Improvement of Gross Theory of Beta-decay and its Application to Nuclear Data

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Present study is the results of "Comprehensive study of delayed-neutron yields for accurate evaluation of kinetics of high-burn up reactors" entrusted to Tokyo Institute of Technology by the Ministry of Education, Culture, Sports, Science and Technology of Japan (MEXT).
Project leader: S. Chiba, **Term**: 2012.11-2016.3, **Budget**: 200M yen (2M USD (1USD=100 yen))
Topic: (1) Fission yield meas.(exp) **(2) Delayed neutron cal. (th)** (3) Fission yield cal. (th) (4) Nuclear Data

Introduction:
Theoretical beta-decay study in nuclear physics

Beta decay : a weak interaction

- **Half-life** : essential quantity. understanding nuclear structure (nuclear matrix element)
- **Delayed neutron from n-rich nuclei** : usage of atomic energy safety, r-process nucleosynthesis
- **Decay heat from actinides** : usage for atomic energy safety
- **Neutrino** : neutrino-nucleus reaction, neutrino from nuclear reactor, etc.

Theoretical approach

- (Configuration) Shell model:
  - K. Langanke, et al. → r-process (neutron-rich)
  - T. Otsuka, et al.
  - A. Brown et. al.,

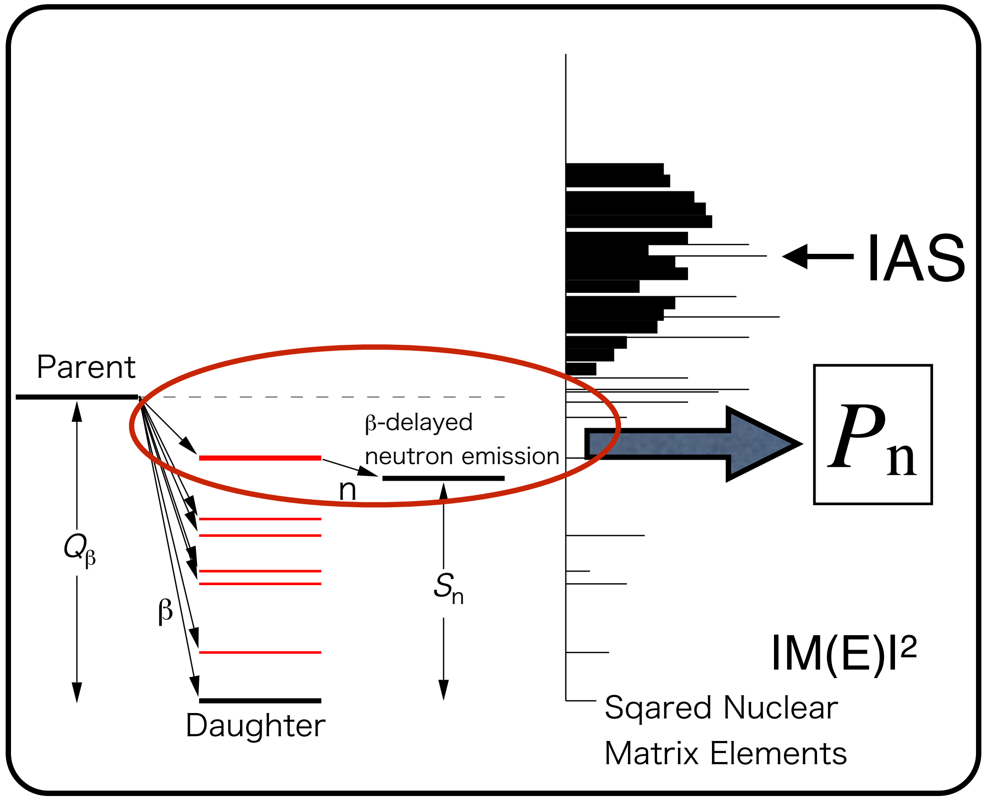
  → light nuclei, double beta decay
- Quasi-particle Random-Phase-Approximation (QRPA):
  - FRDM+QRPA (P. Möller, K.-L. Kratz, et al.)
  - HFB+QRPA (M. Bender, et al., others) →entire region in nuclear chart
  - DF3+QRPA (I. Borzov, et al.)
  - etc.
- Macroscopic approach
  - Gross theory (GT) (T. Tachibana, K. Takahashi, M. Yamada, et al.)

Microscopic ↕ Macroscopic

→ global calculation, half-life, delayed neutron, decay heat, neutrino, etc.

Nuclear β-decay and delayed neutron

Schematic view of beta-decay



Half-life

$$T_{1/2} = \frac{\ln 2}{\lambda}$$

$$\lambda = \lambda_{\mathrm{F}} + \lambda_{\mathrm{GT}} + \lambda_1^{(0)} + \lambda_1^{(1)} + \lambda_1^{(2)}$$ (up to 1st forbidden)

Decay constant

$$\lambda_{\mathrm{F}} = \frac{m_{\mathrm{e}}{}^5 c^4}{2\pi^3 \hbar^7} |g_{\mathrm{V}}|^2 \int_{-Q}^{0} |M_{\mathrm{F}}(E)|^2 f(-E) dE$$

$$\lambda_{\mathrm{GT}} = \frac{m_{\mathrm{e}}{}^5 c^4}{2\pi^3 \hbar^7} |g_{\mathrm{A}}|^2 3 \int_{-Q}^{0} |M_{\mathrm{GT}}(E)|^2 f(-E) dE$$

$$\lambda_1^{(2)} = \frac{m_{\mathrm{e}}{}^5 c^4}{2\pi^3 \hbar^7} \left(\frac{m_{\mathrm{e}} c}{\hbar}\right)^2 |g_{\mathrm{A}}|^2 \int_{-Q}^{0} \sum_{ij} |M_{ij}(E)|^2 f_1(-E) dE$$ <-- unique 1st

$$\lambda_1^{(1)} = \frac{m_{\mathrm{e}}{}^5 c^4}{2\pi^3 \hbar^7} \left(\frac{m_{\mathrm{e}} c}{\hbar}\right)^2 \left[|g_{\mathrm{V}}|^2 \int_{-Q}^{0} |M_{\boldsymbol{r}}(E)|^2 f_{1\mathrm{V}}^{(1)}(-E) dE + |g_{\mathrm{A}}|^2 \int_{-Q}^{0} |M_{\boldsymbol{\sigma}\times\boldsymbol{r}}(E)|^2 f_{1\mathrm{A}}^{(1)}(-E) dE \right]$$

$$\lambda_1^{(0)} = \frac{m_{\mathrm{e}}{}^5 c^4}{2\pi^3 \hbar^7} \left(\frac{m_{\mathrm{e}} c}{\hbar}\right)^2 |g_{\mathrm{A}}|^2 \int_{-Q}^{0} |M_{\boldsymbol{\sigma}\cdot\boldsymbol{r}}(E)|^2 f_{1\mathrm{A}}^{(0)}(-E) dE$$

M(*E*): Nuclear matrix elements
f(-*E*): Integrated Fermi function

Transition type

Trans.	Type	ΔL	Parity ch.
Allowed	Fermi	0	+
	Gamow-Teller	0,±1	+
1st forbid.	non-unique 1st	0,±1	-
	unique 1st	±2	-
2nd forbid.	non-unique 2nd	±2	+
	unique 2nd	±3	+
3rd forbid.	non-unique 3rd	±3	-
	unique 3rd	±4	-

Delayed neutron probability

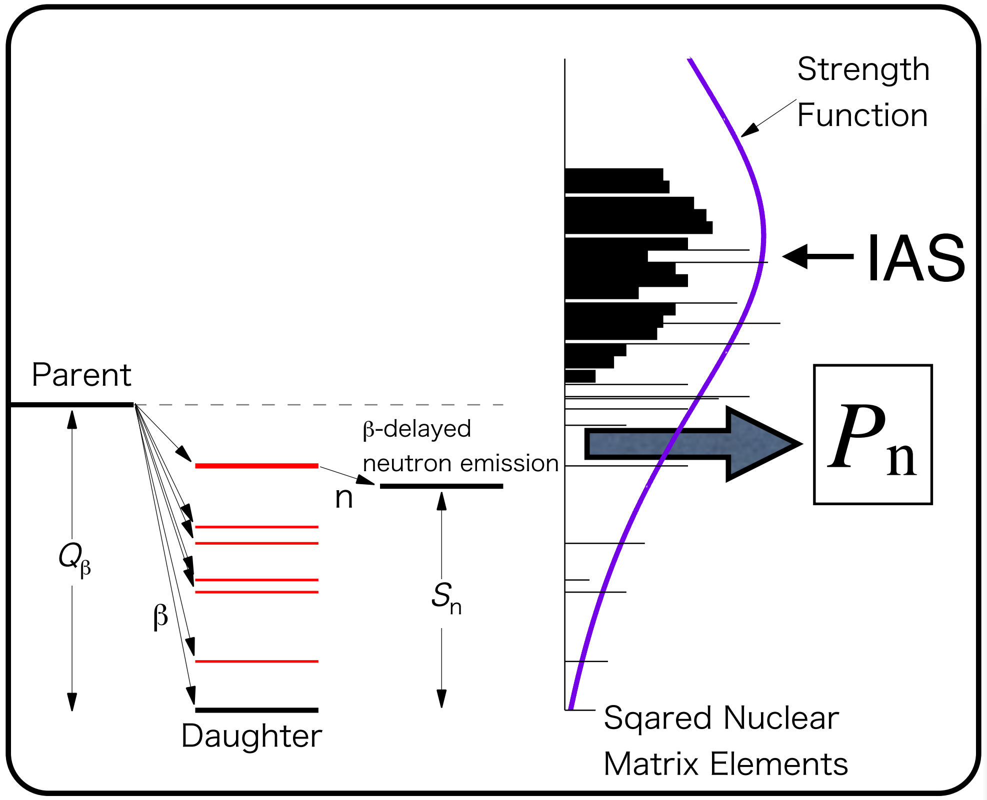
$$P_{\mathrm{n}} = \frac{C}{\lambda_\beta} \int_{-Q_\beta + S_{\mathrm{n}}}^{0} |M(E)|^2 f(Z, -E) \frac{\Gamma_{\mathrm{n}}}{\Gamma_{\mathrm{n}} + \Gamma\gamma} dE$$

decay competition

Delayed neutron is a phenomenon accompanied with the β decay.

Nuclear beta-decay and gross theory

Strength function of beta-decay



Required sum rules

$$\int_{-\infty}^{\infty} D_{\mathrm{F}}(E;\epsilon)dE = 1$$

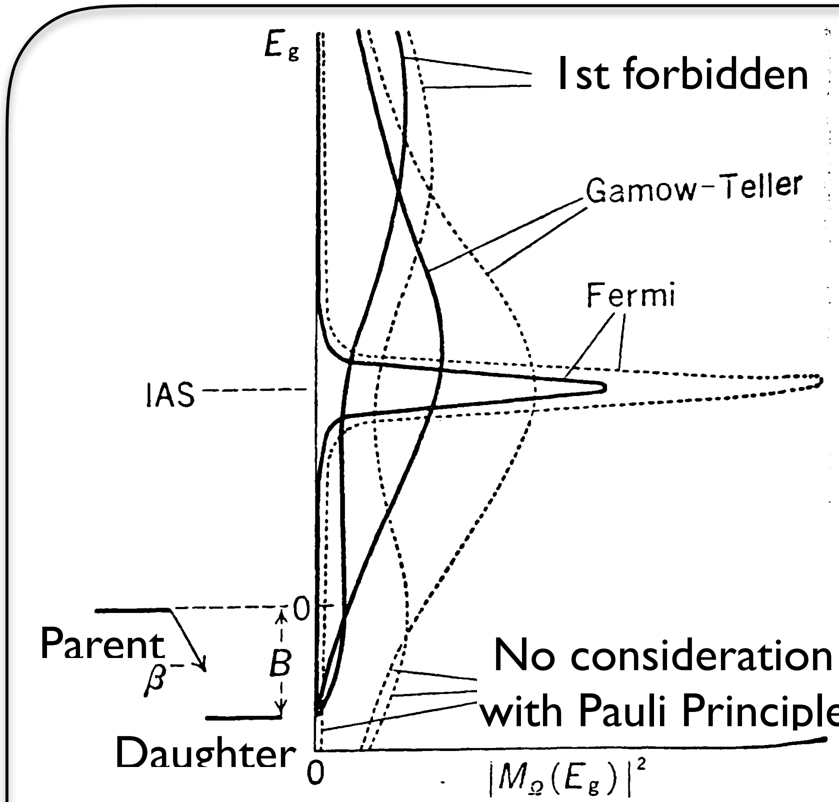
$$\int_{-\infty}^{\infty} D_{\mathrm{GT}}(E;\epsilon)dE = 3$$

$$\int_{-\infty}^{\infty} D_{r}(E;\epsilon)dE = \langle r^2 \rangle \approx \frac{3}{5}R^2$$

$$\frac{1}{3}\int_{-\infty}^{\infty} E D_{\mathrm{GT}}(E;\epsilon)dE = \Delta E_{\mathrm{C}}$$

$$\frac{1}{3}\int_{-\infty}^{\infty} E^2 D_{\mathrm{GT}}(E;\epsilon)dE = (\Delta E_{\mathrm{C}})^2 + \sigma_{\mathrm{C}}^2 + \sigma_{\mathrm{N}}^2$$

Overview of gross theory



$$|M_\Omega(E)|^2 = \int_{\epsilon_{\min}}^{\epsilon_{\max}} D(E,\epsilon)W(E,\epsilon)\frac{dn_1}{d\epsilon}d\epsilon$$

$D(E,\epsilon)$: one particle strength function

$\frac{dn_1}{d\varepsilon}$: level density

K. Takahashi et al., PTP **41** (1969)→Concept
S. Koyama et al., PTP **44** (1970)→$dn_1/d\varepsilon$
K. Takahashi et al., ADNDT **12** (1973)→**GT1**
T. Kondoh et al., PTP **74** (1985)→BCS UV-factor
T. Tachibana et al.. PTP **84** (1990)→D(E, ε)
T. Tachibana et al., Proc. ENAM95 (1995)→**GT2**

Fermi, Gamow-Teller, and 1st forbidden can be calculated.

The gross theory includes:
1. Strength function (sum rules are considered)
2. BCS pairing (simply)
3. Forbidden transition
4. Fermi-gas level density (discrete treatment on the surface level))

Average treatment

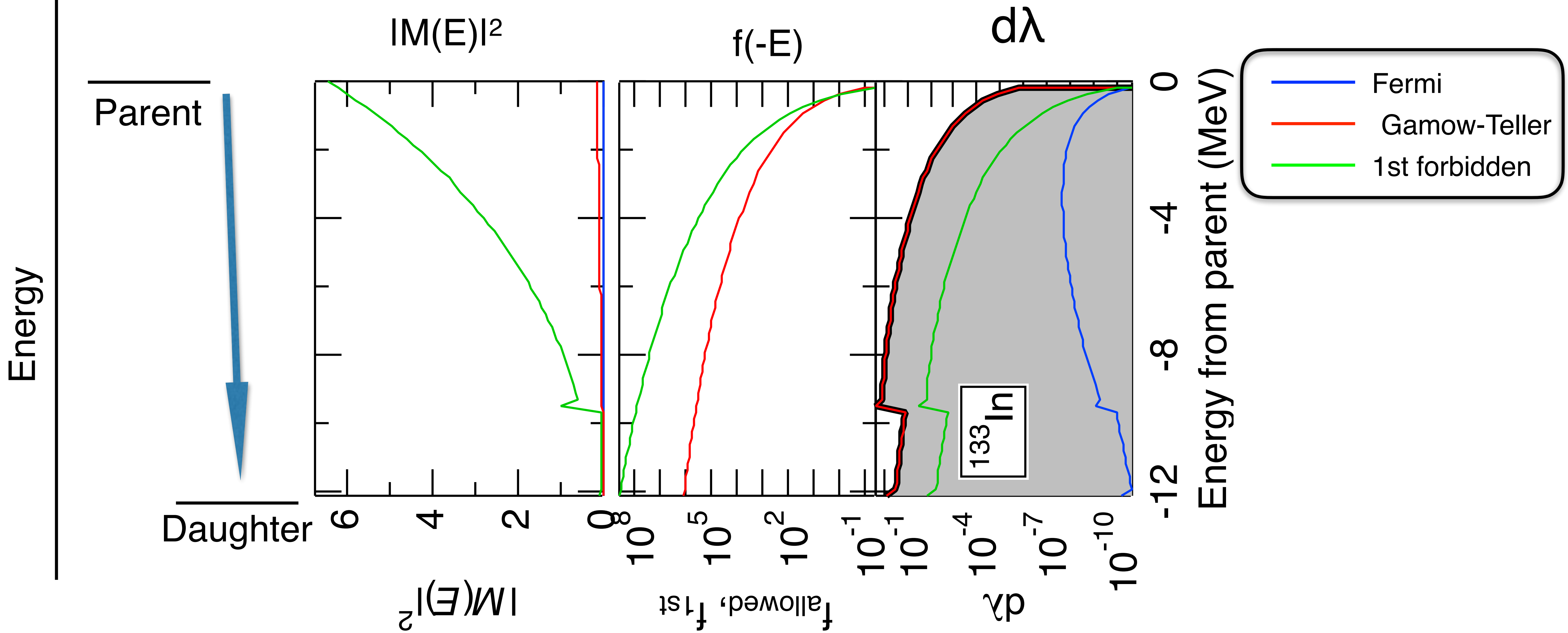
H.K.,S. Chiba, Energy Procedia 71, 228 (2015)

Component of beta-decay calculation

Nuclear matrix elements (Gross theory)

Integrated Fermi (Phase factor of electron)

Intensity



Fermi $$\lambda_{\mathrm{F}} = \frac{m_{\mathrm{e}}{}^5 c^4}{2\pi^3\hbar^7}|g_{\mathrm{V}}|^2 \int_{-Q}^{0} |M_{\mathrm{F}}(E)|^2 f(-E)dE$$

Gamow-Teller $$\lambda_{\mathrm{GT}} = \frac{m_{\mathrm{e}}{}^5 c^4}{2\pi^3\hbar^7}|g_{\mathrm{A}}|^2 3 \int_{-Q}^{0} |M_{\mathrm{GT}}(E)|^2 f(-E)dE$$

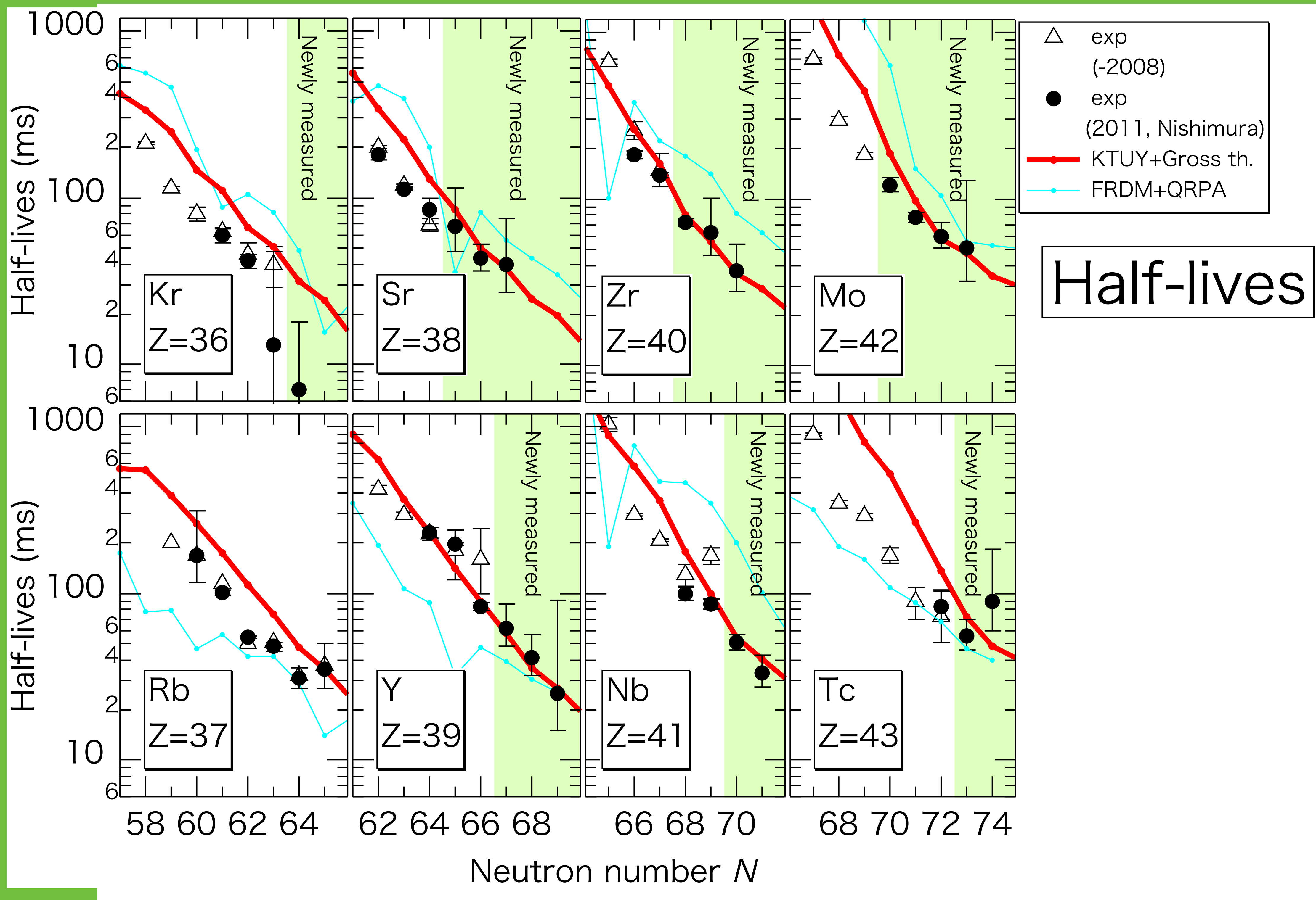
~10^{-7}

1st (unique) $$\lambda_1^{(2)} = \frac{m_{\mathrm{e}}{}^5 c^4}{2\pi^3\hbar^7}\left(\frac{m_{\mathrm{e}}c}{\hbar}\right)^2 |g_{\mathrm{A}}|^2 \int_{-Q}^{0} \sum_{ij} |M_{ij}(E)|^2 f_1(-E)dE$$

<-- unique 1st

Examples of the gross theory

$T_{1/2}$ Systematics of neutron-rich nuclei

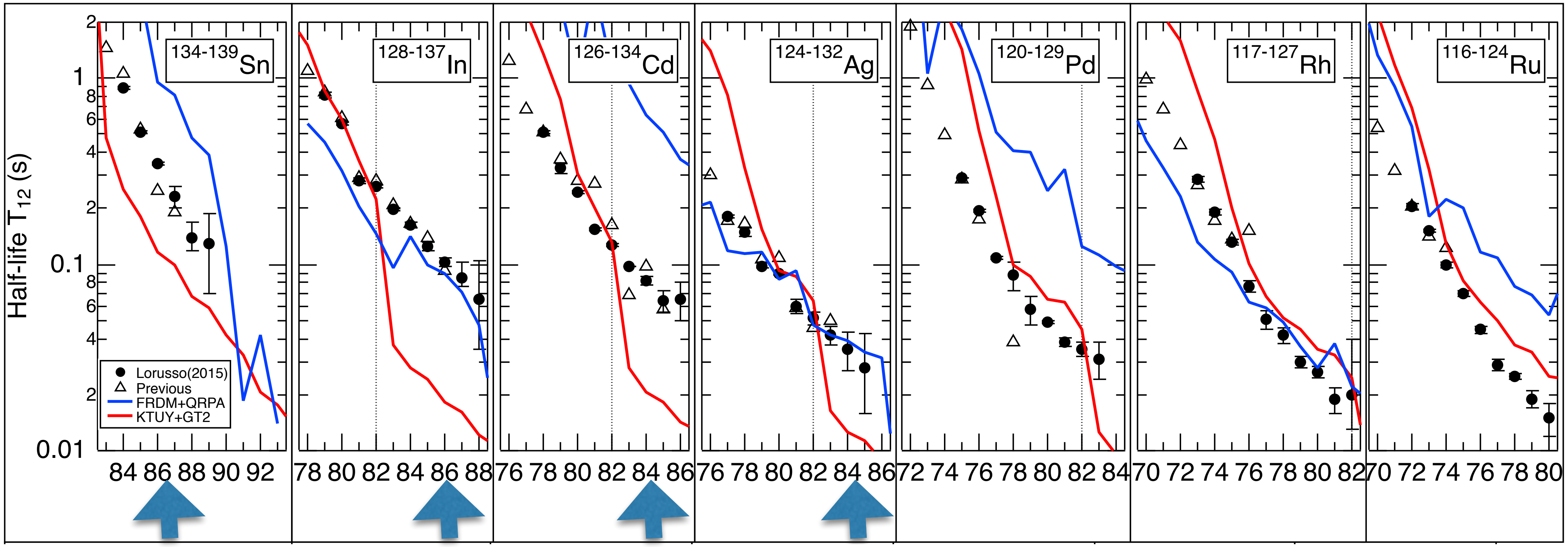


Half-lives

S. Nishimura et al., PRL **106** (2011)

Improvement of the gross theory

Motivation



Neutron number N

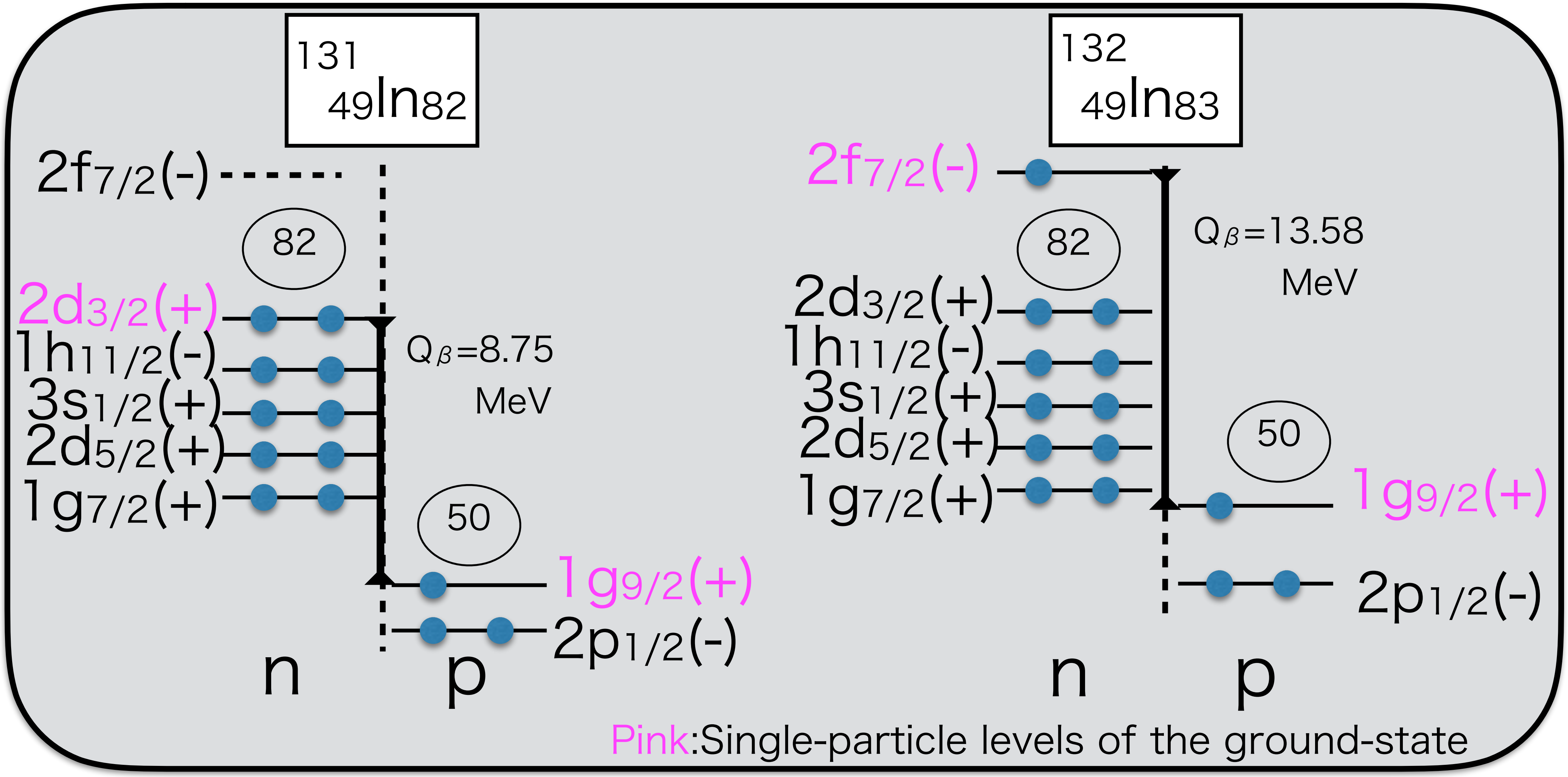
G. Lorusso et al., PRL**114** (2015)

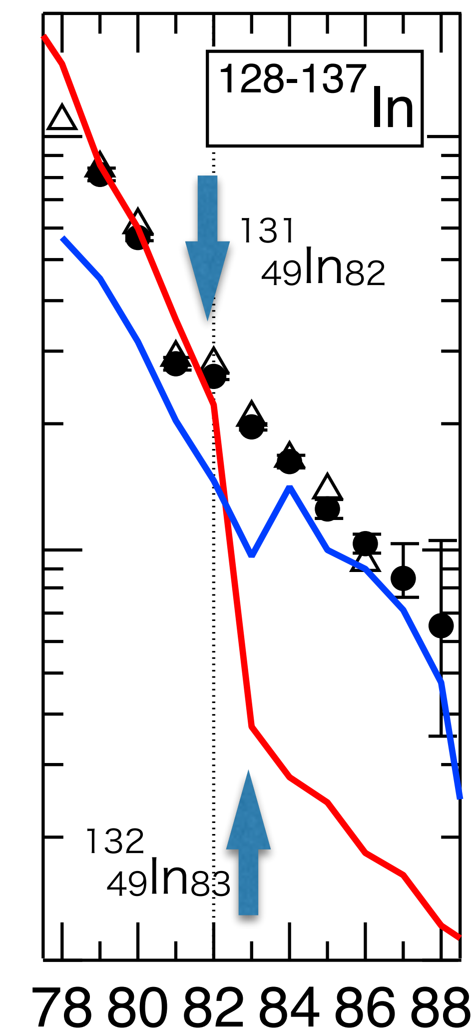
Red : KTUY+GT2
Blue : FRDM+QRPA
●,△ : experiment

Gross theory:
Steep changes in the N=82
(↑)

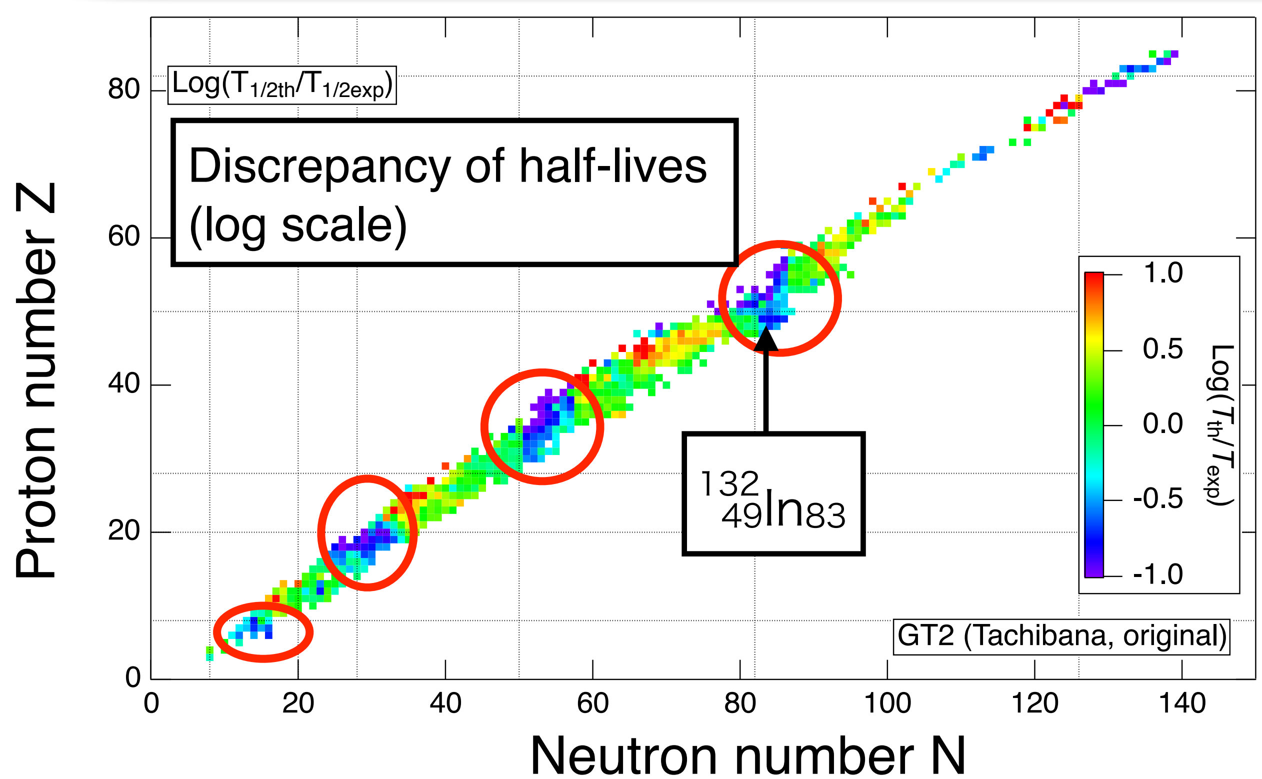
Microscopic treatment has not been done in the gross theory.

Single-particle level



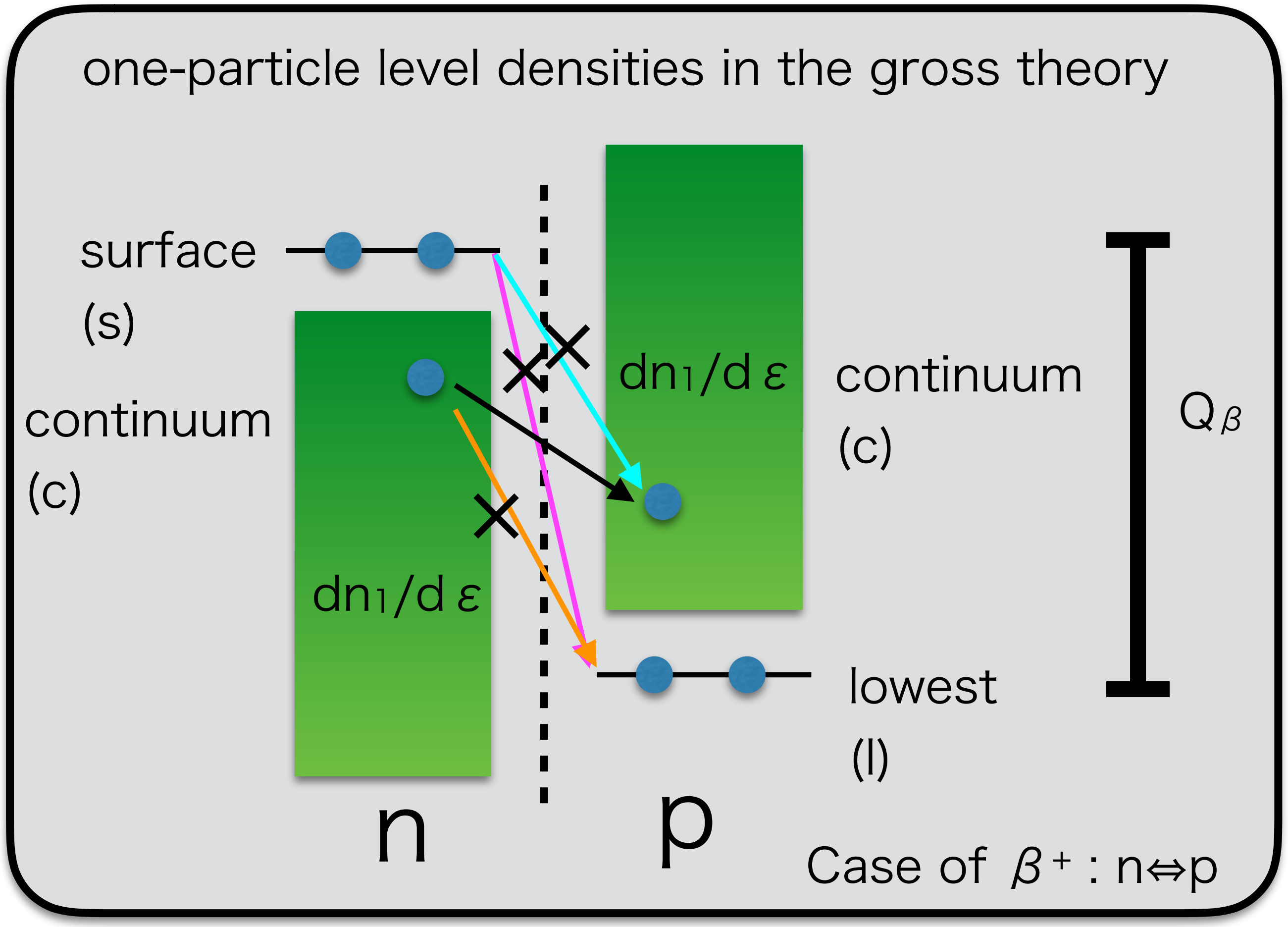


Red : KTUY+GT2
Blue : FRDM+QRPA



Parity change of the ground-state levels(+→- or -→+) occurs.

Method — Improvement of the gross theory



Matrix element of gross theory

$$|M_\Omega(E)|^2 = \int_{\epsilon_{\min}}^{\epsilon_{\max}} D(E,\epsilon)W(E,\epsilon)\frac{dn_1}{d\epsilon}d\epsilon$$

Four components of matrix element

$$|M_\Omega(E)|^2$$

$$= \underbrace{|M_{\Omega S\to l}(E)|^2}_{} + \underbrace{|M_{\Omega S\to C}(E)|^2}_{} + \underbrace{|M_{\Omega C\to C}(E)|^2}_{\text{remain}} + \underbrace{|M_{\Omega C\to l}(E)|^2}_{}$$

suppress

Only apply for **allowed transition**

Procedure

(1)Estimates single-particle spin-parity of nuclei only for small deformed one.

Spin and parity: Woods-saxon potential[2]. Defamation: the KTUY mass model[1]

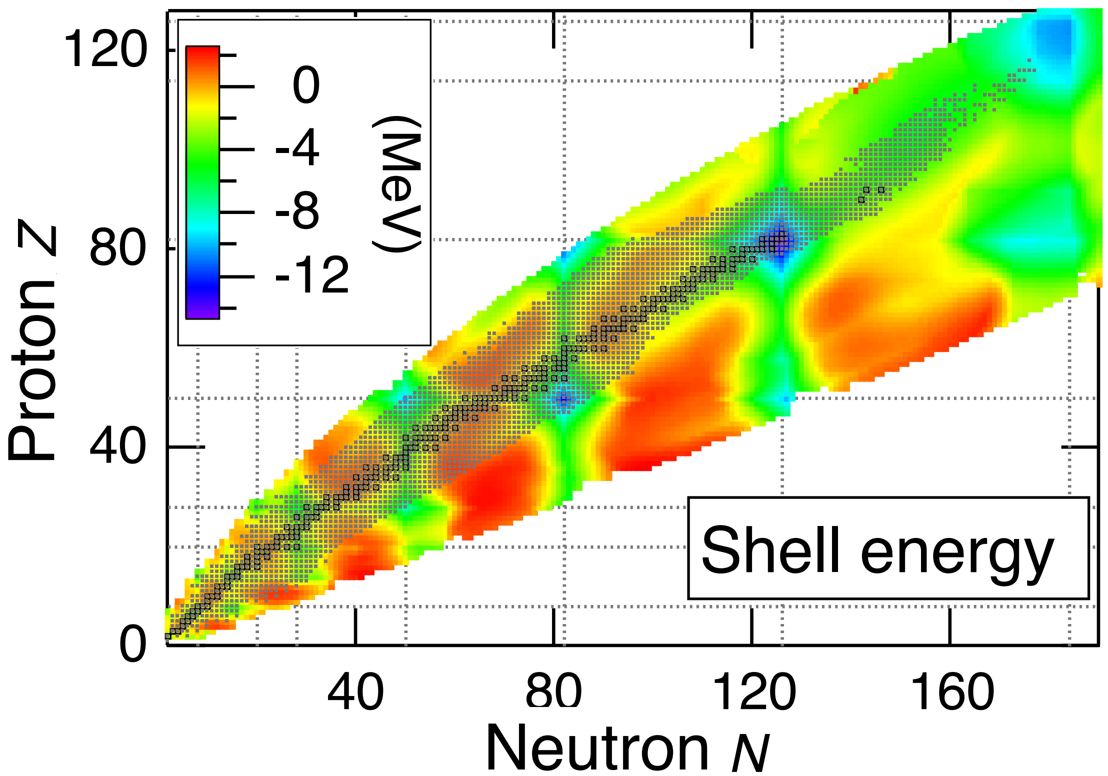
1)H. Koura et al., PTP **113** (2004), 2)H. Koura et al., NPA **671** (2000)

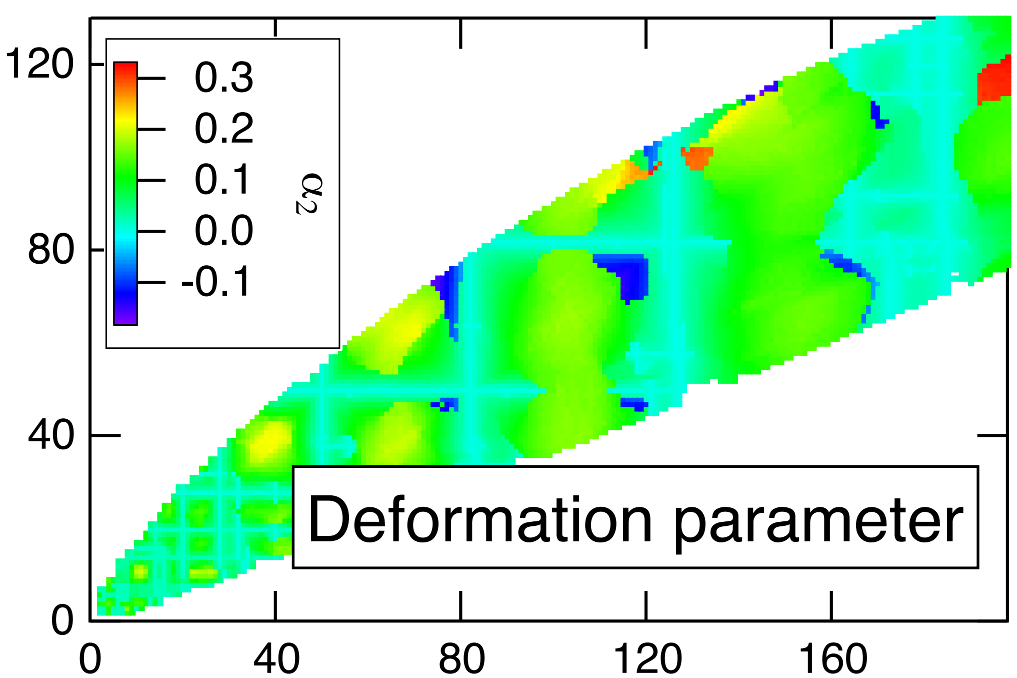
(2)If the parities of neutron and proton is different (+ and -, or - and +)

→suppress the components of the **allowed transition** without the matrix element of $|M_{\Omega C\to C}(E)|^2$

Theoretical models of masses and single-particle levels

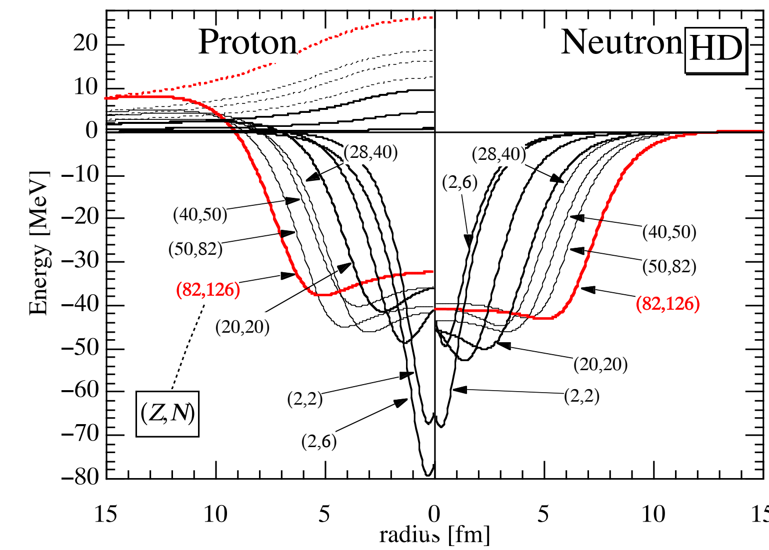
Ground-state masses: spherical-basis model (KTUY)





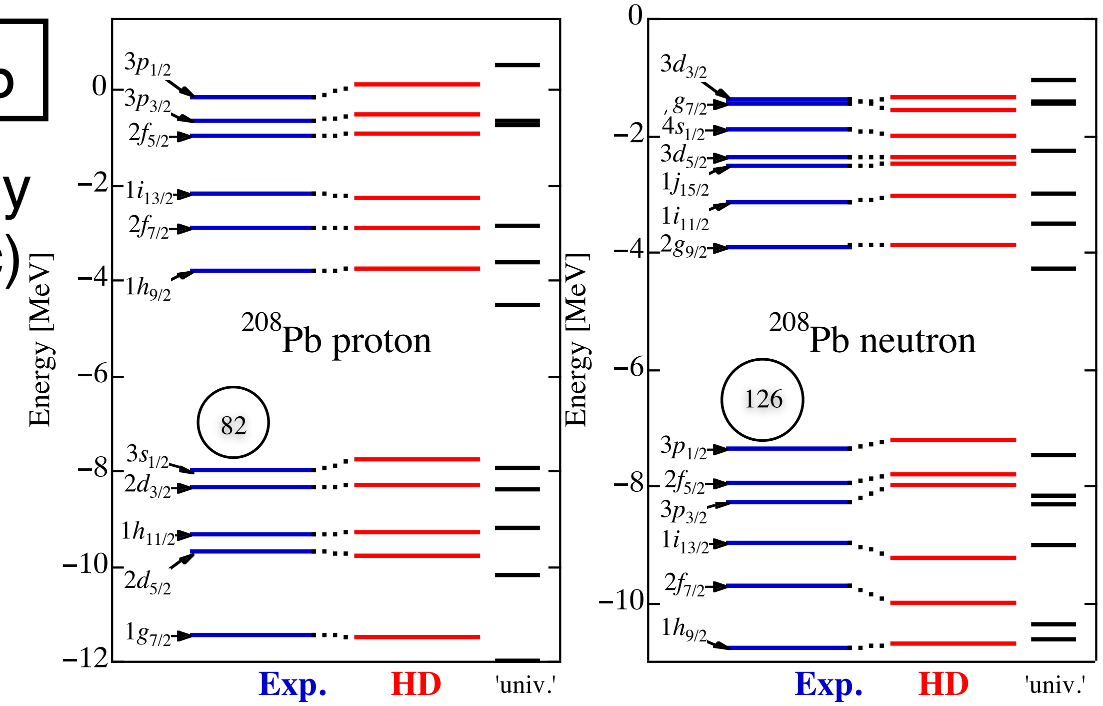
1)H. K., T. Tachibana, et al., PTP **113**, 305(2004)

Single-particle levels: Modified Woods-Saxon pot.



^{208}Pb

(doubly magic)



Within 300 keV
(as absolute values)

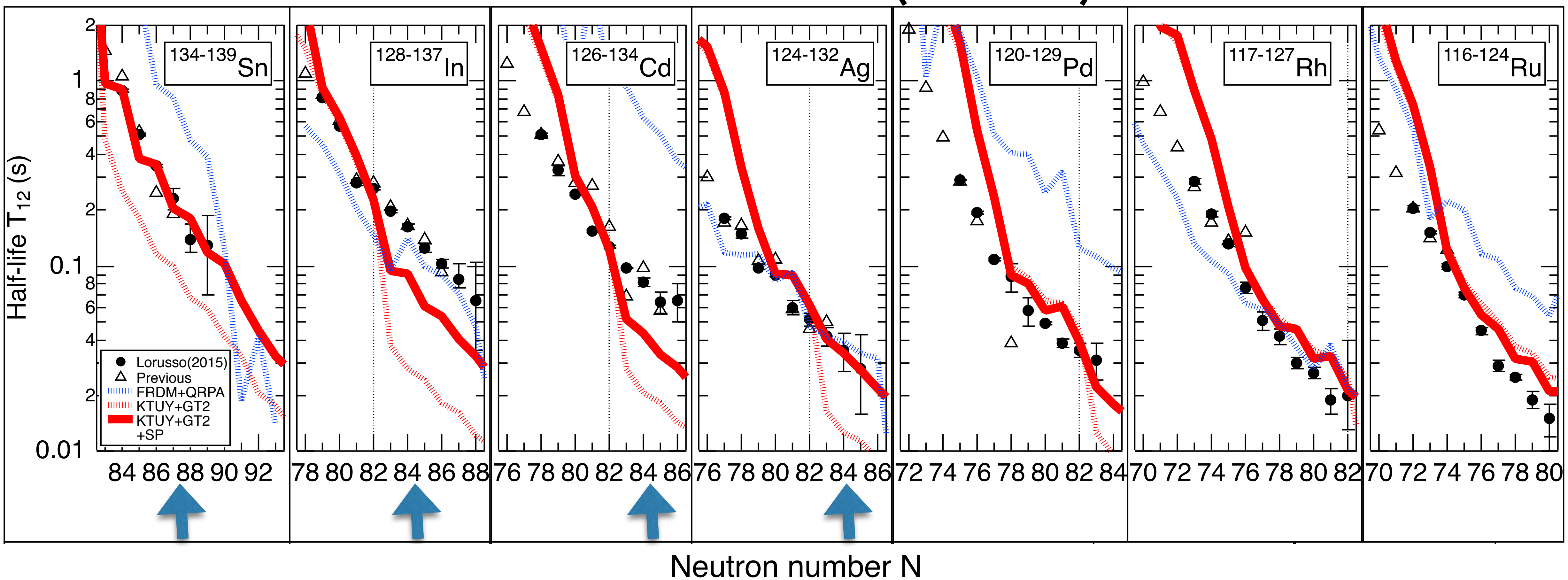
'universal':720 keV

H. K, M. Yamada, NPA **671**, 96(2000)

Adopted as a nuclear data

Results

1:Half-lives (Local)

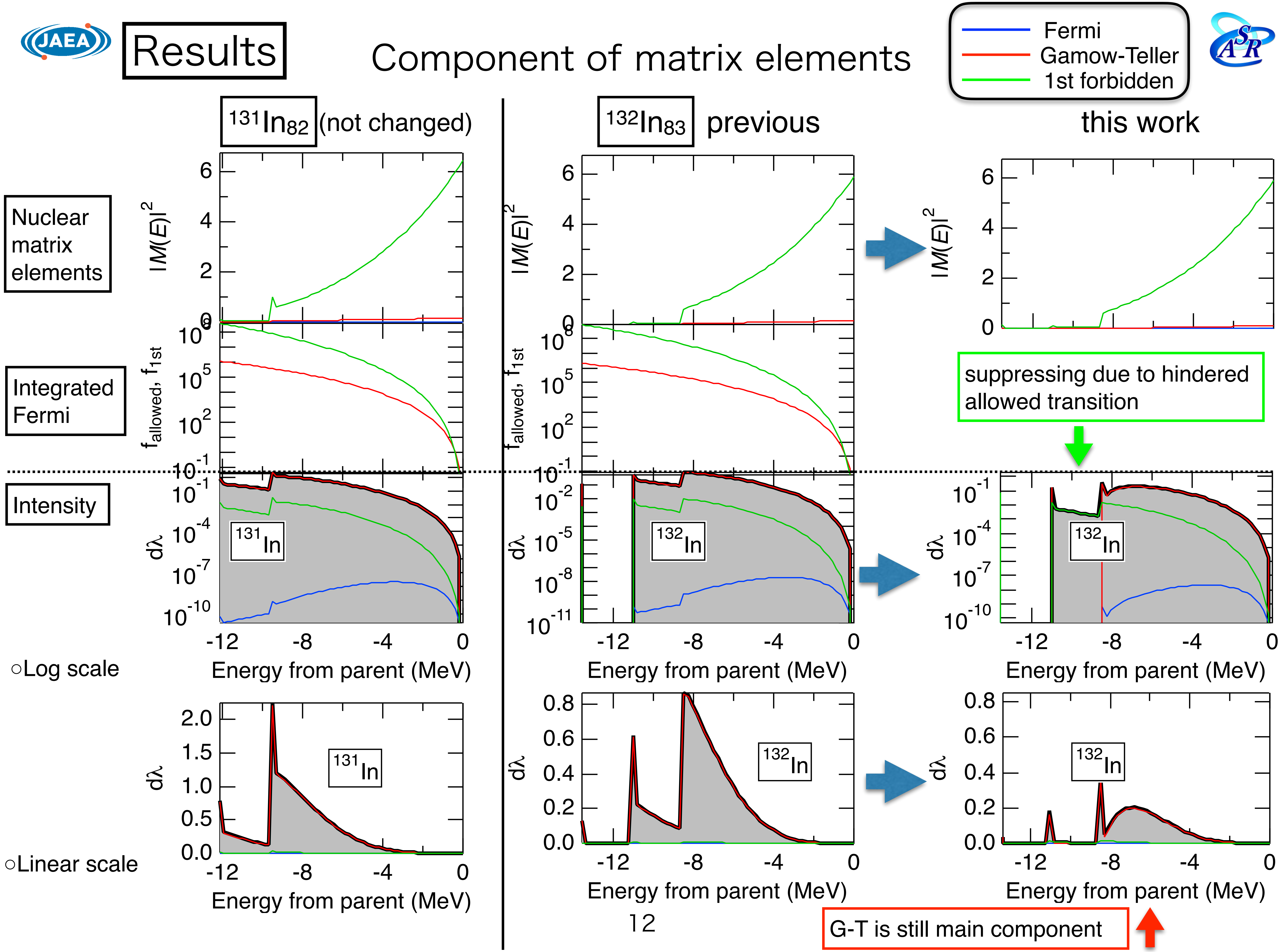


Red : KTUY+GT2
Blue : FRDM+QRPA
RED : KTUY+GT2'
Spin-Parity

Steep changes at N=82 disappear.

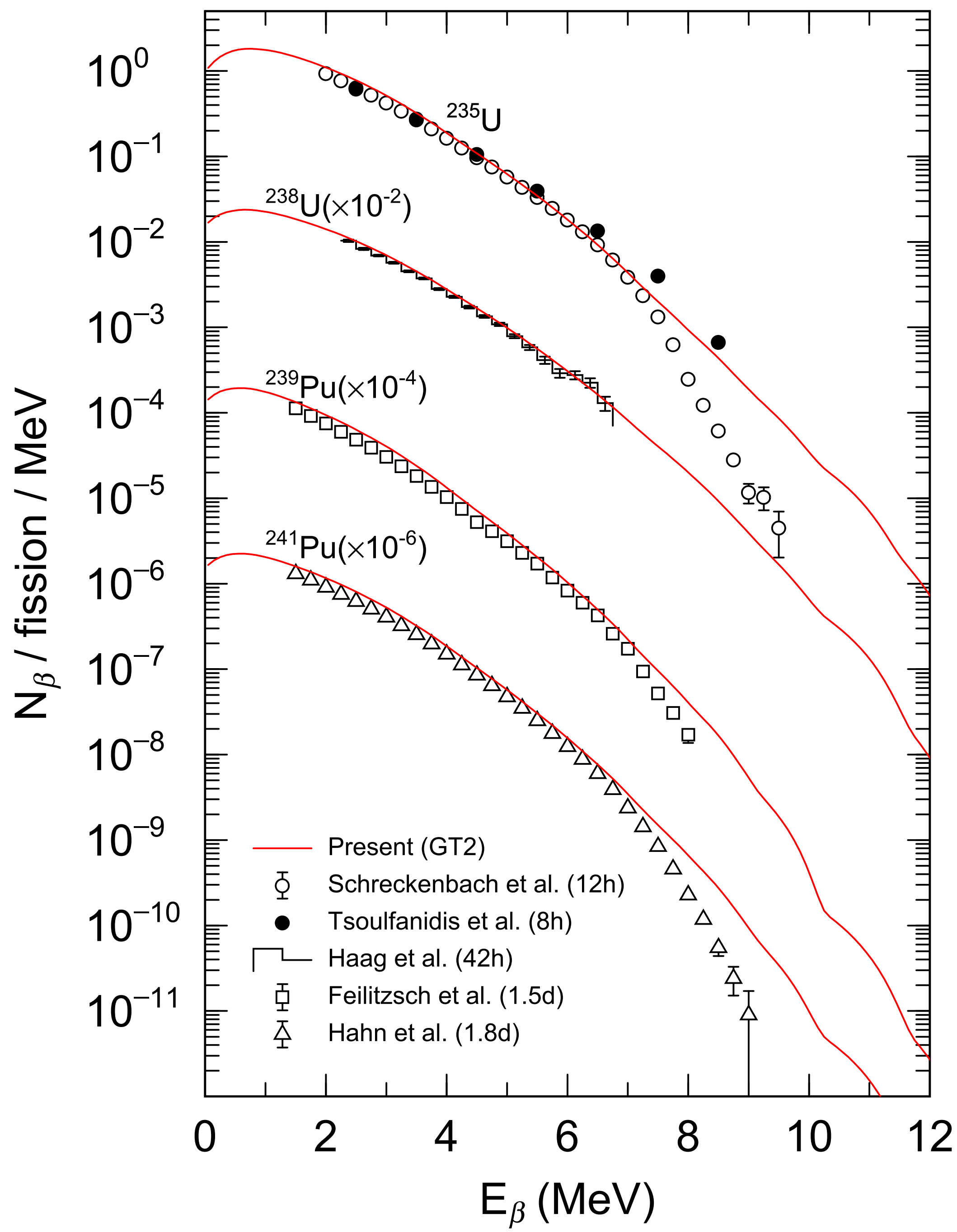
Results

Component of matrix elements



Thursday's Afternoon at Morus: R648 by T.Yoshida

'Analysis of reactor-neutrino spectra fully based on the gross theory …'



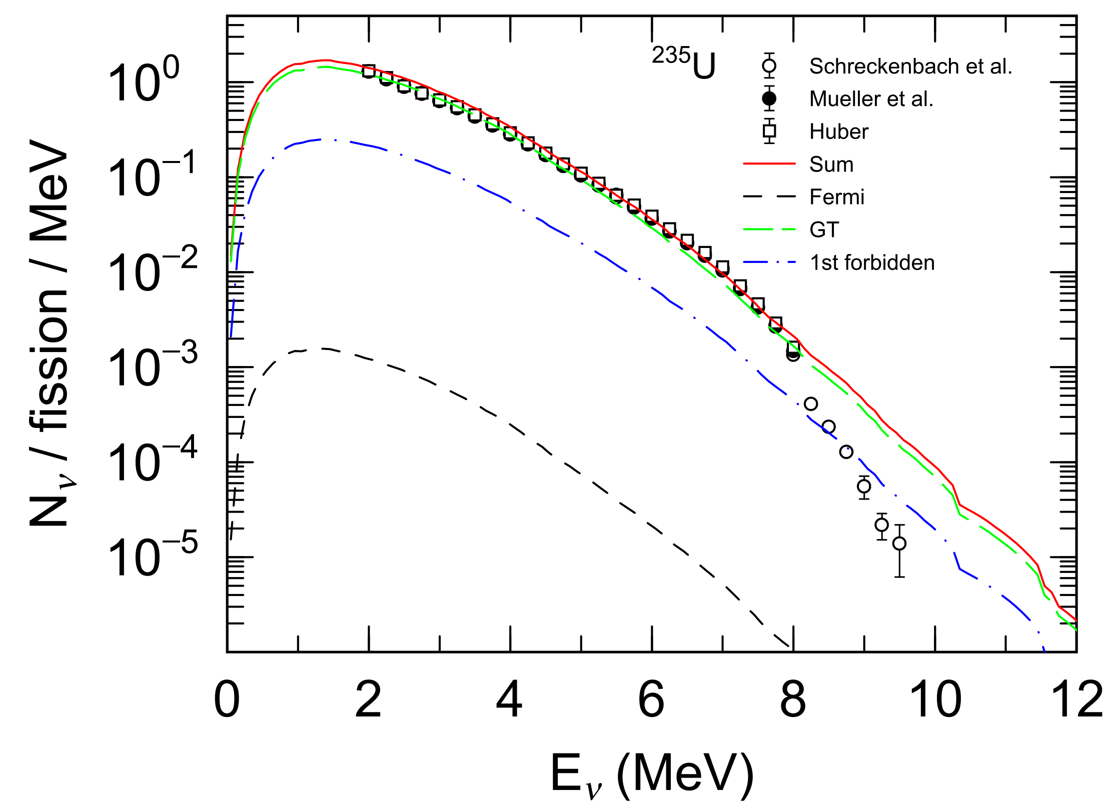


Fig. 2. Calculated antineutrino spectrum from ^{235}U sample and its component-wise breakdown. For reference are shown three spectra converted from the electron spectrum measured by Schreckenbach et al.

Fig. 1. Comparison of the aggregate electron spectra from ^{235,238}U and ^{239,241}Pu samples between the present calculation and the measurements. The data for ^{238}U and ^{239,241}Pu were shifted downward successively by the factors indicated in parentheses for plotting purpose.

T. Yoshida, T. Tachibana, …, S. Chiba, Prog. Nucl. Ener. **88**, 175-181 (2016)

Conclusion

- Gross theory of beta-decay is improved on the nuclear structure:
  Parity-mismatching is considered for the allowed transition

- Discrepancies of half-lives from experiments are systematically improved

- Programming codes is developed as a project on delayed neutron study

- Beta decay and accompanying processes are calculated with various versions of the gross theory.