Effects of microscopic transport coefficients on fission observables calculated by Langevin equation and its systematics

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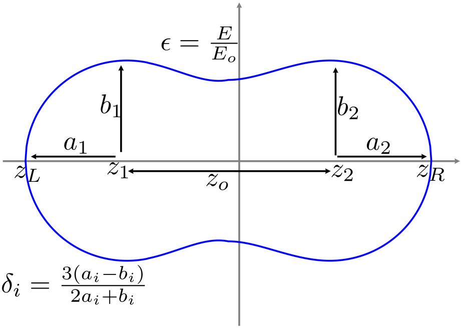
Background

- Fission observables obtained by Dynamical description with Langevin equation employing **macroscopic transport equation** studied for example by
  - Asano, PhD. thesis, Konan University (2004).
  - Aritomo et al., PRC88 044614(2013), PRC90 054609(2014).
  - Pahlavani and Mirfathi, PRC92, 024622(2015).
- Microscopic transport coefficient had previously been used among others by
  - Omsk-Dubna-Kiev group in Langevin equation for **fusion-fission (Quasi-fission)**
    - Kosenko et. al., J. Nucl. Radiochem. Sci. 3, pp 71 (2002)
    - Litnevsky et al. PRC89, 034626 (2014)
  - Adamian et. al. (microscopic mass tensor diagonal component) for **fusion** description in di-nuclear system, Nuc. Phys. A 646, pp 29 (1999) and PRC91,054602 (2015)
- Primary objective is to extend the work done in Usang(2016, PRC in press)
- We will validate our Langevin calculation result for low energy fission using microscopic transport coefficient with:
  - JENDL-4.0: Fission Product Yield
  - Flynn et. al. 1974: Heavy and light fission fragments mass average
  - Viola 1966,1985: Total Kinetic Energy (TKE) Systematics
  - Exp data from Dyachenko 1968, Zoeller 1991, Yanez 2014: TKE dependence on excitation energy

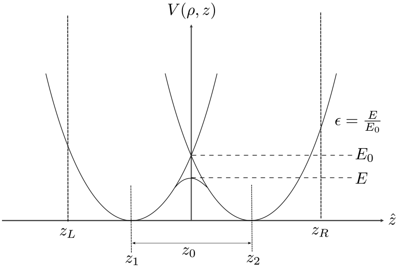
Shape parameterization



where we use the parameter,

$$\delta_i = \frac{3\,(a_i - b_i)}{2a_i + b_i}; \quad Z_{z0} = \frac{z_o}{R}; \quad \alpha = \frac{A_1 - A_2}{A_1 + A_2}; \quad \frac{a_i}{b_i} = \frac{\omega_{\rho_i}}{\omega_{z_i}} = \frac{1 - \frac{2}{3}\delta_i}{1 + \frac{1}{3}\delta_i}; \quad \epsilon = 0.35$$

Two Centre Deformed Nuclear Potential



$$f_1(z, z_i) = 1 + c_i(z - z_i) + d_i(z - z_i)^2;$$
$$f_2(z, z_2) = 1 + g_i(z - z_i)^2$$
$$z_1 = -z_o \frac{\omega_{z2}}{\omega_{z_1} + \omega_{z_2}}; \quad z_2 = z_o \frac{\omega_{z1}}{\omega_{z_1} + \omega_{z_2}};$$
$$c_1 = c_2 = 2 - 4\epsilon; \quad d_1 = d_2 = 1 - 3\epsilon$$
$$g_1 = (1 - Q^2)\frac{z_1}{z_o}; \quad g_2 = Q^2(1 - Q^2)\frac{z_2}{z_o};$$
$$Q = \frac{\omega_{\rho_1}}{\omega_{\rho_2}}; \quad \alpha = \frac{V_L - V_R}{V_L + V_R}$$

Potential ($\mathbf{L}.\mathrm{s}$ and ℓ^2 term not written here) from Maruhn & Greiner (1972):

$$V(\rho, z) = \begin{cases} \frac{1}{2}m\omega_{z_1}^2(z - z_1)^2 + \frac{1}{2}m\omega_{\rho_1}^2\rho^2; & z \le z_1 \\ \frac{1}{2}m\omega_{z_1}^2(z - z_1)^2 f_1(z, z_1) + \frac{1}{2}m\omega_{\rho_1}^2\rho^2 f_2(z, z_1); & z_1 \le z \le 0 \\ \frac{1}{2}m\omega_{z_2}^2(z - z_2)^2 f_1(z, z_2) + \frac{1}{2}m\omega_{\rho_2}^2\rho^2 f_2(z, z_2); & 0 \le z \le z_2 \\ \frac{1}{2}m\omega_{z_2}^2(z - z_2)^2 + \frac{1}{2}m\omega_{\rho_2}^2\rho^2; & z_2 \le z \end{cases}$$

Linear Response Theory

- Ehrenfest Theorem
  (Given $E = \mathrm{tr}(\hat{\rho}\hat{H})$)

$$\boxed{\frac{dE_{total}}{dt} = 0}$$

$$\Rightarrow \left\langle \frac{\partial \hat{H}}{\partial Q} \right\rangle \frac{dQ}{dt} = 0$$

- Secular Equation

$$\boxed{\kappa^{-1} + \tilde{\chi}(\omega) = 0}$$

- Equation of motion

$$\boxed{M \frac{d^2 q(t)}{dt^2} + \gamma \frac{dq(t)}{dt} + Cq(t) = 0}$$

with local approx., $q = Q - Q_m$

- Coefficient: Mass, Friction and Stiffness tensor

$$M = \frac{1}{2} \left.\frac{d^2\tilde{\chi}}{d\omega^2}\right|_{\omega=0} ; \quad \gamma = -i \left.\frac{d\tilde{\chi}}{d\omega}\right|_{\omega=0}$$

$$C = -\tilde{\chi}(\omega = 0) - \kappa^{-1}$$

$$-\kappa^{-1} = \left\langle \left.\frac{d^2\hat{H}}{dQ^2}\right|_{Q_0} \right\rangle ; \; (m^{-1})_{ij} = \frac{1}{M}$$

- Response Function,

$$\boxed{\tilde{\chi}(\omega) = \sum_{jk} \tilde{F}_{jk}\tilde{F}_{kj} \frac{n_k - n_j}{\hbar\omega - (\varepsilon_k - \varepsilon_j) + i\epsilon}}$$

- Collective Response tensor,

$$\boxed{\boldsymbol{\chi}_{coll}(\omega) = \boldsymbol{\kappa}(\boldsymbol{\kappa} + \boldsymbol{\chi}(\omega))^{-1}\boldsymbol{\chi}(\omega)}$$

Microscopic Mass and Friction Tensor

$$\gamma_{\mu\nu}(0) = 2\hbar \sum_{jk}{}' (n_k^T - n_j^T)\xi_{kj}^2 \frac{E_{kj}^- \Gamma_{kj}}{\left[(E_{kj}^-)^2 + \Gamma_{kj}^2\right]^2} F_\mu^{kj} F_\nu^{jk}$$
$$+ 2\hbar \sum_{jk} \left(n_k^T + n_j^T - 1\right) \eta_{kj}^2 \frac{E_{kj}^+ \Gamma_{kj}}{[(E_{kj}^+)^2 + \Gamma_{kj}^2]^2} F_\mu^{kj} F_\nu^{jk} \,,$$

$$M_{\mu\nu}(0) = \hbar^2 \sum_{jk}{}' (n_k^T - n_j^T)\xi_{kj}^2 \frac{E_{kj}^{-2}[E_{kj}^- - 3\Gamma_{kj}]}{[(E_{kj}^-)^2 + \Gamma_{kj}^2]^3} F_\mu^{kj} F_\nu^{jk}$$
$$+ \hbar^2 \sum_{jk} \left(n_k^T + n_j^T - 1\right) \eta_{kj}^2 \frac{E_{kj}^{+2}[E_{kj}^+ - 3\Gamma_{kj}]}{[(E_{kj}^+)^2 + \Gamma_{kj}^2]^3} F_\mu^{kj} F_\nu^{jk} \,.$$

Given quasi-particle states, E_k, E_j in BCS and Bogolyubov-Valatin coefficients u_k, v_k, we define as in Ivanyuk(2000):

$$E_{kj}^- \equiv E_k - E_j \,, \qquad E_{kj}^+ \equiv E_k + E_j \,, \qquad n_k^T \equiv 1/(1 + \exp(E_k/T)) \,,$$
$$\eta_{kj} = u_k v_j + u_j v_k \,, \qquad \xi_{kj} = u_k u_j - v_k v_j \,.$$

Langevin Process I

$$\frac{dq_i}{dt} = \left(m^{-1}\right)_{ij} p_j$$
$$\frac{dp_i}{dt} = -\frac{\partial V}{\partial q_i} - \frac{1}{2}\frac{\partial}{\partial q_i}\left(m^{-1}\right)_{ij} p_j p_k - \gamma_{ij}\left(m^{-1}\right)_{jk} p_k + g_{ij} R_j(t)$$

q_i: deformation, $\{q\}_{3D} = (Z_{z0}, \delta, \alpha)$ and $\{q\}_{4D} = (Z_{z0}, \delta_1, \delta_2, \alpha)$

p_i: momentum conjugate to q_i

m_{ij}: mass tensor (Hydrodynamical mass, LRT mass)

γ_{ij}: friction tensor (Wall and window, LRT friction)

$g_{ij} R_j(t)$: Random force

$R_j(t)$: white noise, R where $\langle R_j(t)\rangle = 0$, $\langle R_i(t_1) R_j(t_2)\rangle = 2\delta_{ij}\delta(t_1 - t_2)$

g: strength of random force where $\sum_k g_{ik} g_{jk} = T\gamma_{ij}$ (Einstein Relation)

T: Temperature

E^*: Excitation Energy

a: level density parameter

E_{int}: Intrinsic Energy where,

$$\boxed{E_{int} = E^* - \frac{1}{2}\left(m^{-1}\right)_{ij} p_i p_j - V(q,T) = aT^2}$$

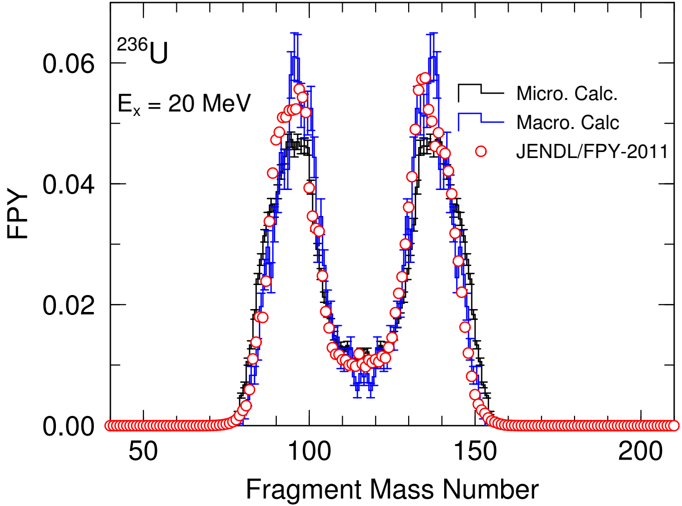
V: Potential energy where,

$$\boxed{V(q,T) = V_{macro}(q) + \Phi_{sh}(E_{int})V_{sh}(q,T=0) + \Phi_{pair}(E_{int})V_{pair}(q,T=0)}$$

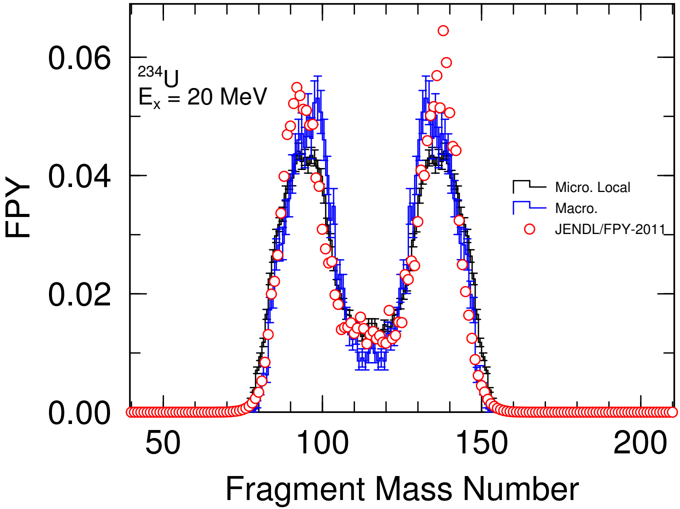
$\Phi(E^*)$: Shell and pair correction factor from (Randrup & Möller, 2013) where,

$$\boxed{\Phi(E^{int}) = \frac{1 + e^{\frac{E_1}{E_o}}}{e^{\frac{E^*}{E_o}} + e^{\frac{E_1}{E_o}}}}$$

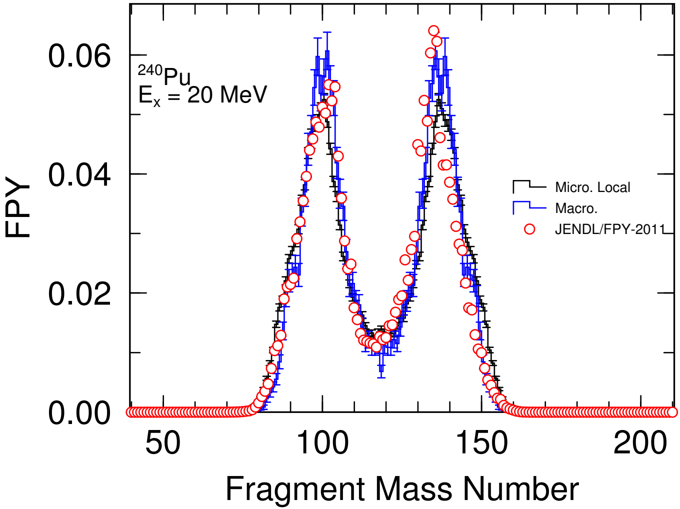
U-236 Compound Nucleus



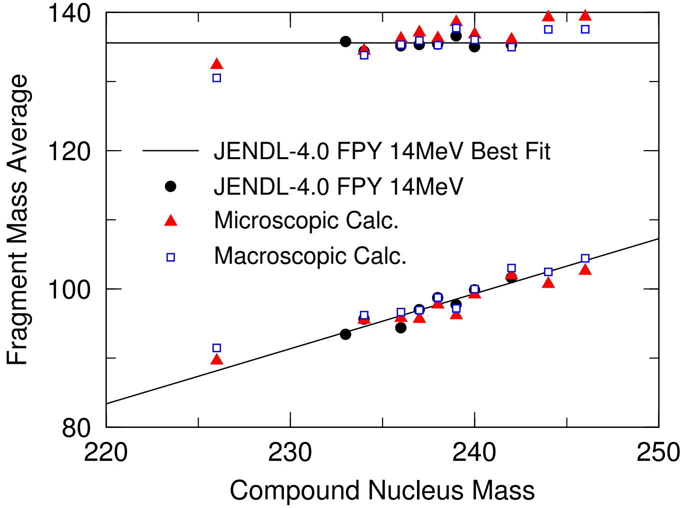
U-234 Compound Nucleus



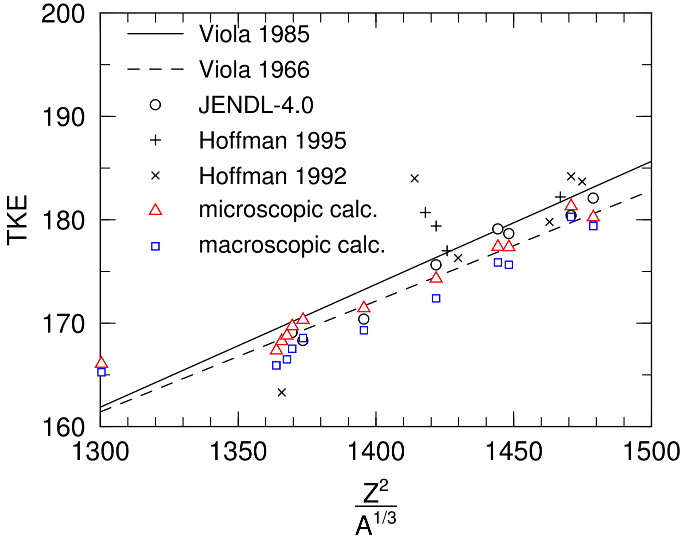
Pu-240 Compound Nucleus



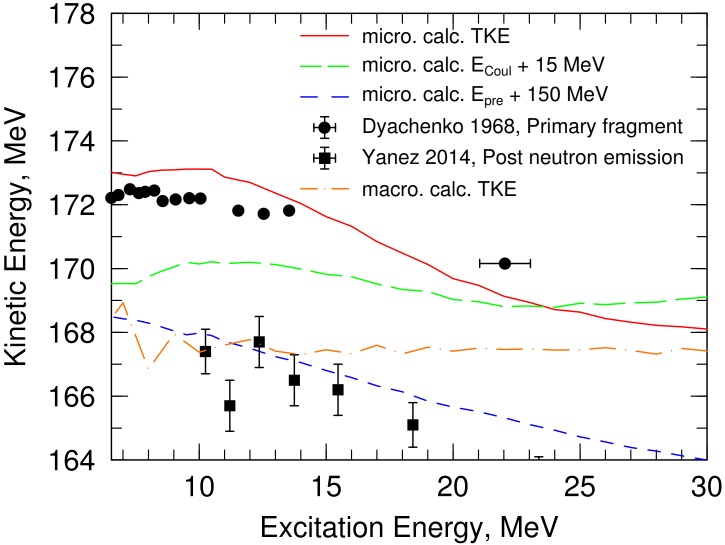
Mass Average Distribution



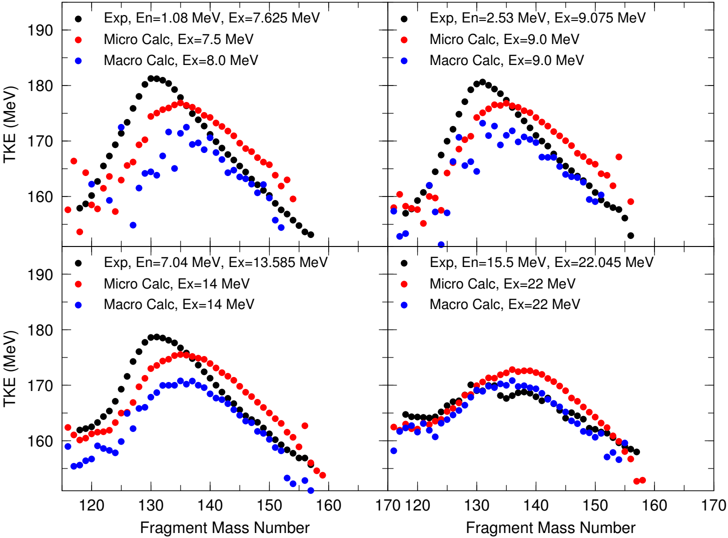
TKE Distribution



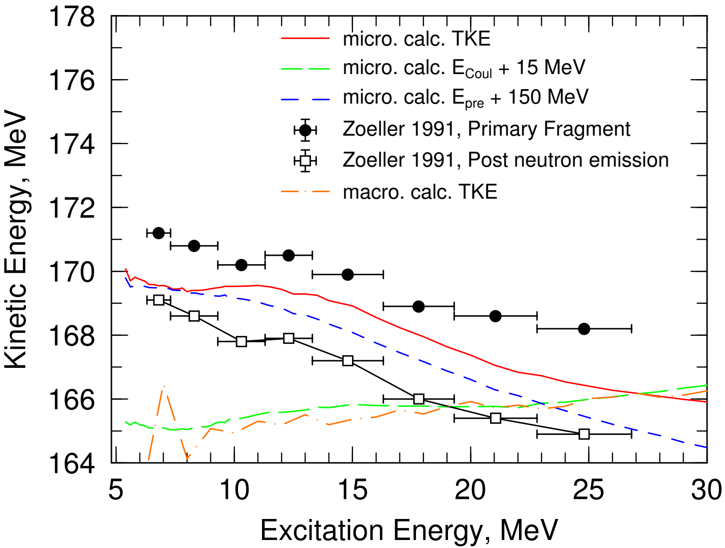
U-236 Compound Nucleus



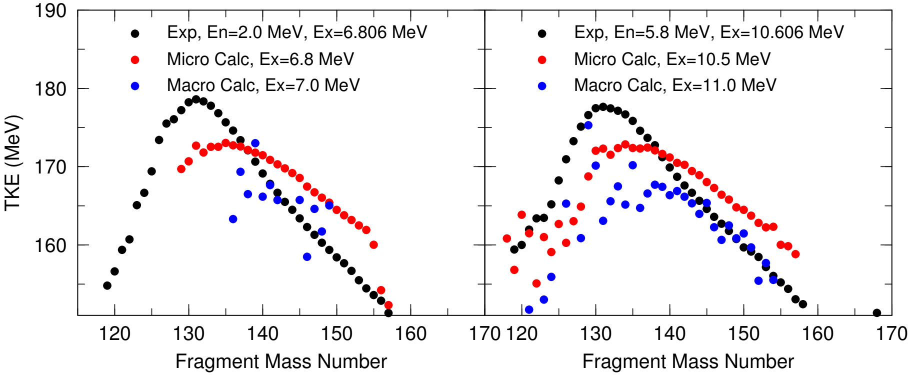
U-236 Compound Nucleus



U-239 Compound Nucleus



U-239 Compound Nucleus



Conclusion

Langevin description of fission with microscopic transport coefficients:

1. obtain satisfactory agreement of fission yield distributions to JENDL-4 FPY
2. close agreement to heavy and light fragment mass average systematics
3. close agreement to Viola's TKE systematics
4. with micro. calc. TKE showing clear dependence on excitation energy.
5. with macro. calc. TKE are independent of excitation energy.
6. Micro. calc. E_{Coul} fluctuates but seems independent of increasing E_x
7. Micro. calc. E_{pre} decays with respect to E_x and
8. Decay of micro. calc. TKE with respect to E_x is due to the decreasing E_{pre} at higher E_x.