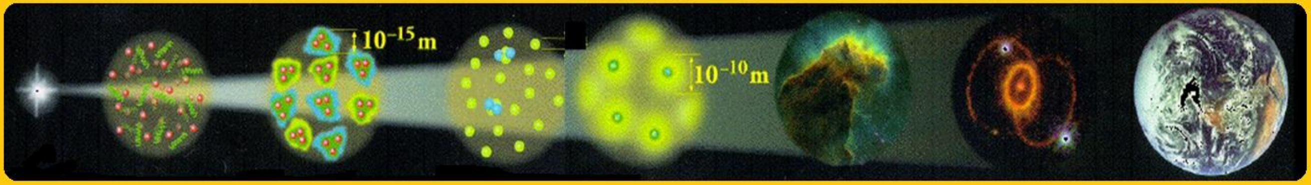

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International Conference on Nuclear Data for science and technology

11-16 Sep. 2016, Bruges, Belgium

Role of Different Nuclear Charge Radii Parameterizations on the Thermal Equilibrium in Nuclear Reactions

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Outline:

- **Introduction:**
  - **Nuclear charge radii parameterizations**
  - **Heavy Ion Collisions (HICs)**
- **Methodology: IQMD model**
- **Motivation**
- **Result and Discussion**
- **Conclusion**
- **Acknowledgments**

NEA NUCLEAR ENERGY AGENCY

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IAEA International Atomic Energy Agency Atoms for Peace

Introduction

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Various forms of nuclear charge radius: (Isospin-independent)

Liquid drop model (LDM)

$$R = r_o A^{1/3}, \text{where} \quad r_o \approx 1.12\, fm$$

Bass (1973)

$$R = 1.07 A^{1/3}$$

Bass (1977)

$$R = 1.233 A^{1/3} - 0.978 A^{-1/3}$$

Christen and Winther (1976)

$$R = 1.233 A^{1/3} - 0.978 A^{-1/3}$$

Broglia and Winther (1991)

$$R = 1.233 A^{1/3} - 0.98 A^{-1/3}$$

Proximity (1977)

$$R = 1.28 A^{1/3} - 0.76 + 0.8 A^{-1/3}$$

Age Winther (1995)

$$R = 1.20 A^{1/3} - 0.09$$

Introduction

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Various forms of nuclear charge radius: (Isospin-dependent)

By H. Ngo and Ch. Ngo
(NGO)

$$R = \frac{(1.1375 + 1.875 \times 10^{-4}) N A^{1/3} + 1.128 Z A^{1/3}}{A}$$

H. Ngo and Ch. Ngo, Nucl. Phys. A **348**, 140 (1980).

By Pomorska and Pomorski
(PP)

$$R = 1.240 A^{1/3} (1 + \frac{1.646}{A} - 0.191 \frac{N - Z}{A})$$

B. Nerlo-Pomorska, K. Pomorski, Z. Phys. A **344**, 359 (1993).

By Royer and Rousseau
(RR)

$$R = 1.2332 A^{1/3} + \frac{2.8961}{A^{2/3}} - 0.18688 A^{1/3} I$$

G. Royer and R. Rousseau, Eur. Phys. J. A **42**, 541 (2009).

And there are many more in the literature: I. Dutt and R. K. Puri, Phys. Rev. C **81, 064608 (2010);** ***ibid. 81, 064609 (2010); ibid. 81,*** 044615 (2010); *ibid.* ***81, 047601 (2010); ibid. 83, 047601 (2011).***

Heavy Ion Collisions (HICs)

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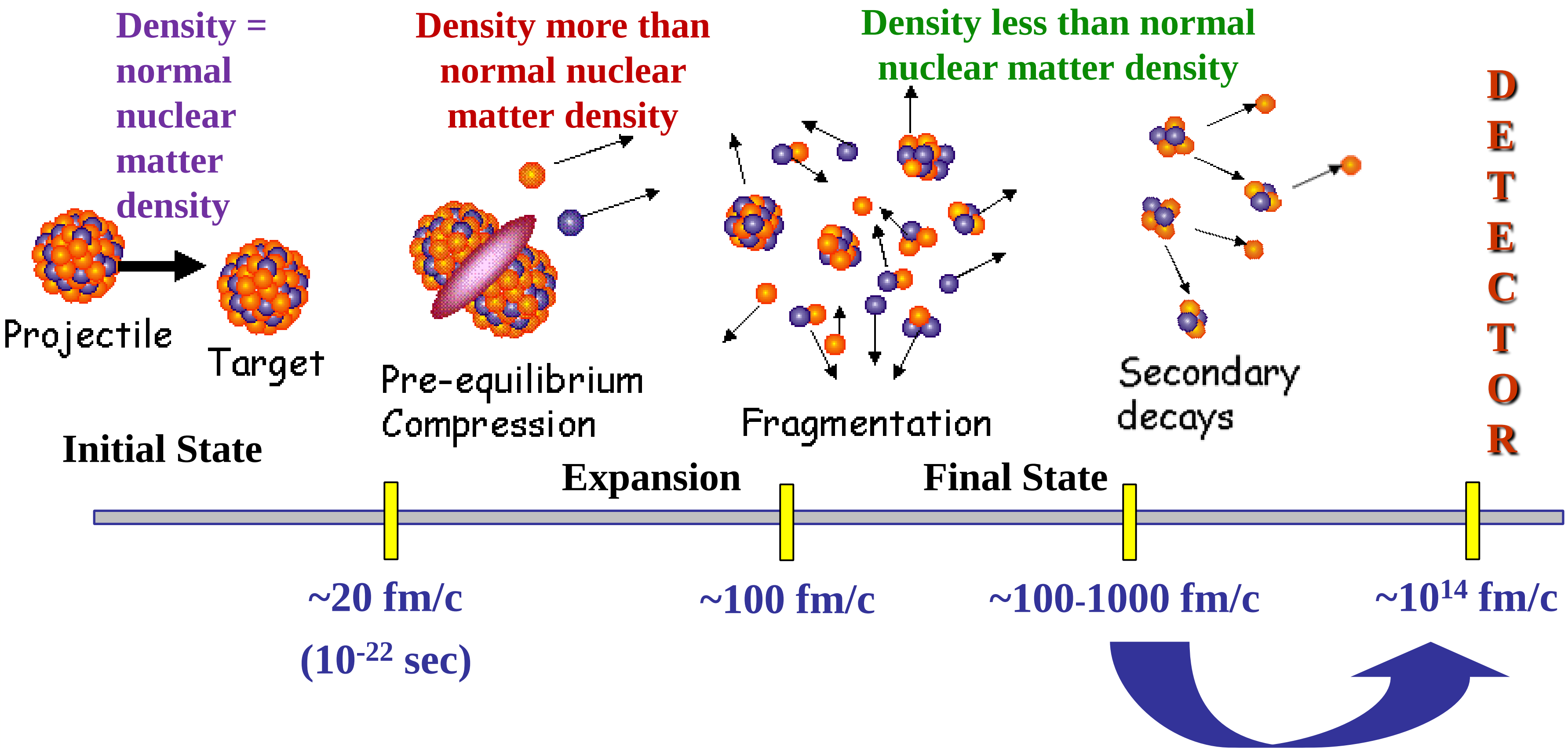
Incident energy = 30 MeV/nucleon – 2 GeV/nucleon



Heavy Ion Collisions (HICs)

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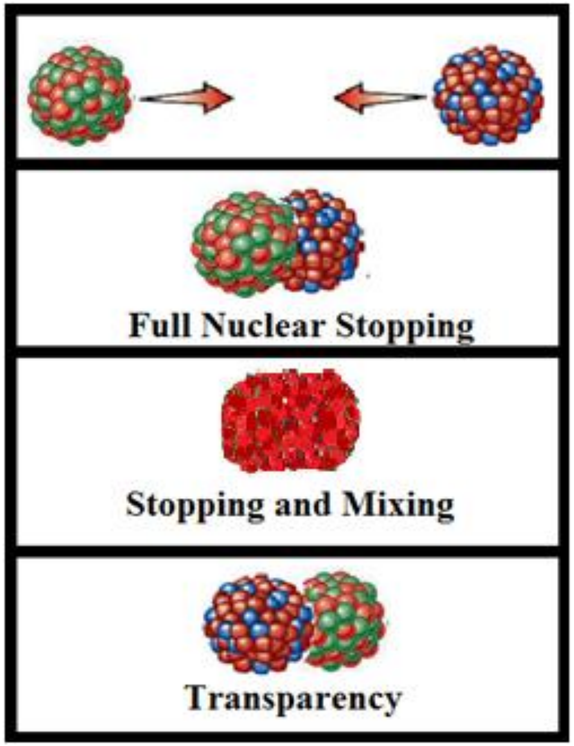
Nuclear Stopping

Nuclear stopping measures the stopping strength of the participant zone of the projectile and target nuclei.

The amount of nuclear stopping determines the energy and volume of the interaction region, which govern the reaction dynamics and the extent to which conditions might be favourable for formation of a high density, de-confined phase of matter.



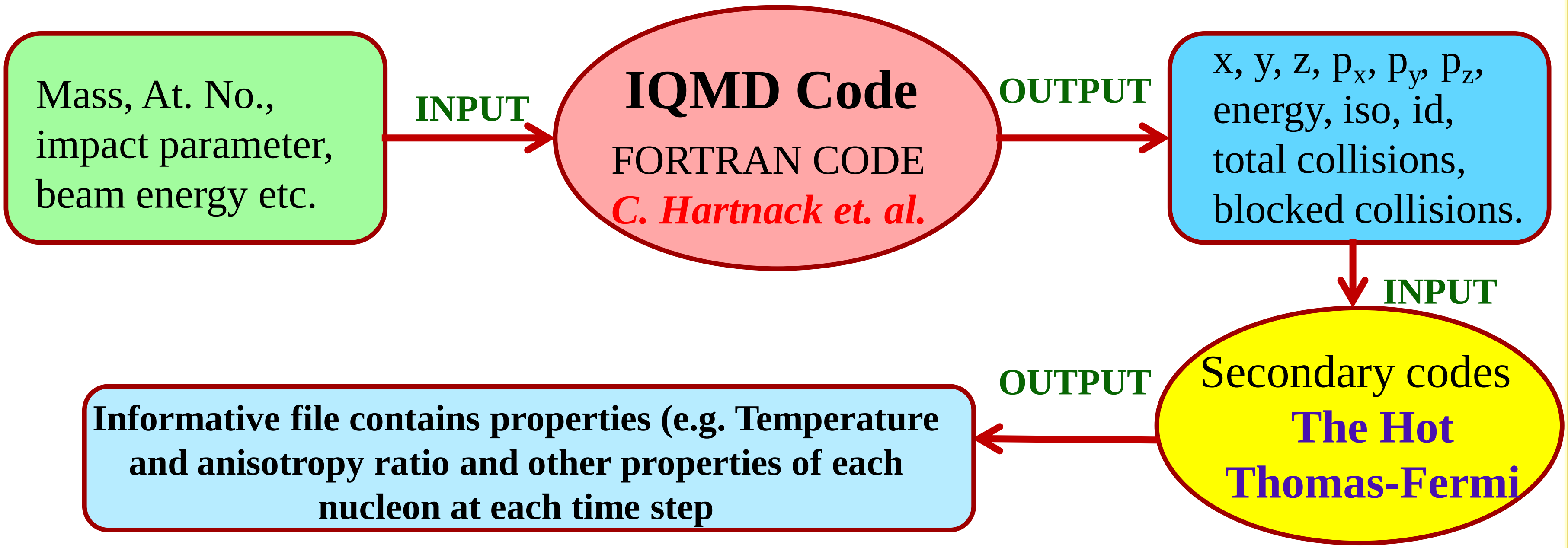
Anisotropic Ratio:

$$R = \frac{2}{\pi}\frac{\sum_i |p_\perp(i)|}{\sum_i |p_\parallel(i)|}, \textit{Where} \quad p_\perp = \sqrt{p_x^2(i) + p_y^2(i)}, \quad p_\parallel = p_z(i)$$

Methodology

Flow Chart of Simulations:

Primary Models to Generate Phase Space of Nucleons



Secondary Models to study the Thermalization

D. T. Khoa *et al., Nuclear Physics A 548, 102 (1992); R.K. Puri et al., Nucl. Physics A* 575, 733 (1994).

Methodology

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Isospin-dependent molecular dynamics (IQMD) model:

START

↓

Initialization

First one has to generate the nuclei.

↓

Propagation

nucleons then propagate under the influence of mean field.

↓

Collision

Finally, nucleons are bound to collide if they come too close to each other.

↓

STOP

Giving input parameters:

- **Mass of projectile and target**
- **Beam energy**
- **Number of events**
- **Number of time steps**
- **Equation of state (Soft EOS and Hard EOS)**
- **Radius parameter**
- **Cross-section parameter**
- **Switches for:**
  - **Various potential ON/OFF**
  - **Delta decay ON/OFF**

C. Hartnack et. al., Eur. Phys. Journal A 1, 151 (1998).

C. Hartnack et al., Phys. Rep. 510, 119 (2012)

Methodology

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Isospin-dependent molecular dynamics (IQMD) model:

START

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Initialization

First one has to generate the nuclei.

↓

Propagation

nucleons then propagate under the influence of mean field.

↓

Collision

Finally, nucleons are bound to collide if they come too close to each other.

↓

STOP

- **Loop over number of events**

Baryons are represented by Gaussian-shaped density distributions:

$$f_i(\vec{r},\vec{p},t)=\frac{1}{(\pi\hbar)^3}e^{-[\vec{r}-\vec{r}_i(t)]^2\frac{1}{4L}}.e^{-[\vec{p}-\vec{p}_i(t)]^2\frac{2L}{\hbar^2}}$$

Nucleons are initialized in a sphere with radius, in accordance with the Liquid Drop Model (LDM).

$$R_{LDM}=R_oA^{1/3}\ fm$$

This can be replaced by any other form of radius.

C. Hartnack et. al., Eur. Phys. Journal A 1, 151 (1998).

C. Hartnack et al., Phys. Rep. 510, 119 (2012)

Methodology

Isospin-dependent molecular dynamics (IQMD) model:

START

↓

Initialization

First one has to generate the nuclei.

↓

Propagation

nucleons then propagate under the influence of mean field.

↓

Collision

Finally, nucleons are bound to collide if they come too close to each other.

↓

STOP

The centroid of each nucleon propagate under the classical Hamiltonian's equation of motion:

$$\frac{dr_i}{dt} = \frac{\partial \langle H \rangle}{\partial p_i}; \quad \frac{dp_i}{dt} = -\frac{\partial \langle H \rangle}{\partial r_i}$$

$$,Where \ \langle H \rangle = \langle T \rangle + \langle V \rangle$$

$$\langle H \rangle = \sum_i \frac{p_i^2}{2m_i} + \sum_i \sum_{j>i} \int f_i(\vec{r}, \vec{p}, t) \, V^{ij}(\vec{r}, \vec{r}') \times f_j(\vec{r}', \vec{p}', t) \, dr \; dr' \, dp \; dp'$$

The baryon-baryon potential is given by:

$$V^{ij} = V^{ij}_{Skyrme} + V^{ij}_{Yukawa} + V^{ij}_{Coul} + V^{ij}_{mdi} + V^{ij}_{sym}$$

C. Hartnack et. al., Eur. Phys. Journal A 1, 151 (1998).

C. Hartnack et al., Phys. Rep. 510, 119 (2012)

Methodology

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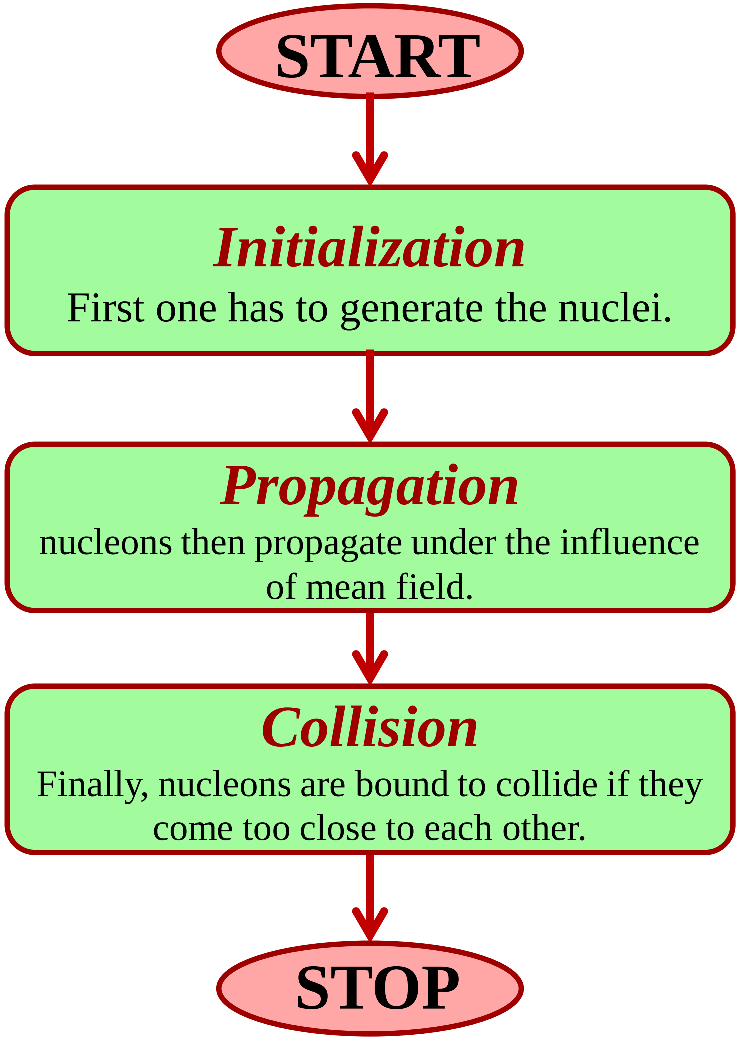
Isospin-dependent molecular dynamics (IQMD) model:



Two particles collide if the minimum relative distance of the centroids of the Gaussians during their motion, in their centre of mass frame fulfils the requirement:

$$\left|r_i - r_j\right| \leq \sqrt{\frac{\sigma_{tot}}{\pi}}, \sigma_{tot} = \sigma\left(\sqrt{s}, type\right)$$

" type" denotes the ingoing collision partners (N-N,N-Δ,N-π,..)

C. Hartnack et. al., Eur. Phys. Journal A 1, 151 (1998).

C. Hartnack et al., Phys. Rep. 510, 119 (2012)

Motivation

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Knowledge about the nuclear parameters including nuclear charge radii is of great interest for low density and low energy exotic phenomena like neutron skin (difference between the root mean square (rms) radius of neutron and proton) thickness, halo nuclei, understanding nuclear size, nucleon distribution and nucleon-nucleon interactions inside the nucleus.

Observables of HICs at intermediate energies are also expected to be influenced by different radii parameterizations of nuclear charge radius.

The additional isospin dependence through nuclear structure may alter the isospin physics.

Result and Discussion

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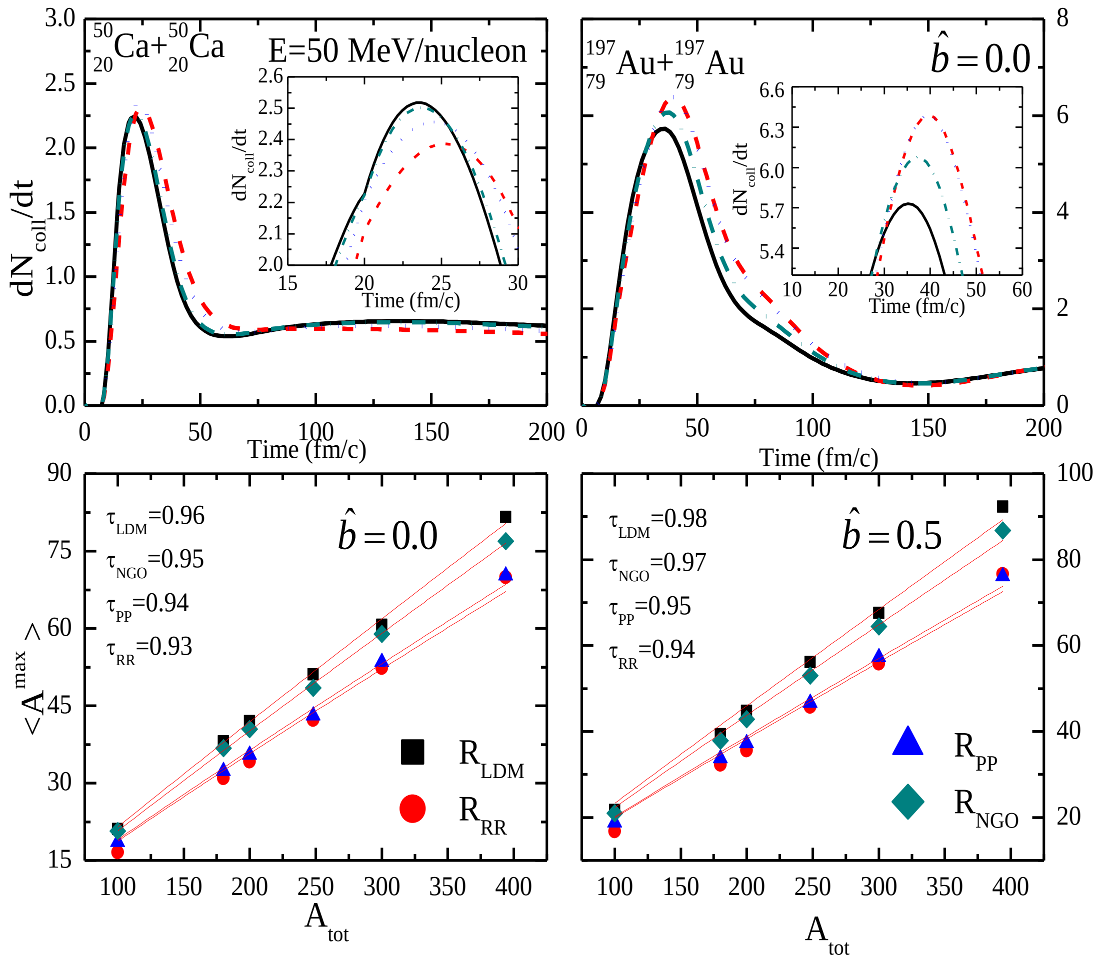
Structural effects via nuclear radius:



$$R_{LDM} > R_{NGO} > R_{PP} > R_{RR}$$

The percentage increase in radius due to different isospin dependent nuclear charge radii parameterizations with respect to R_{LDM} are:

System mass effects:

$$A^{max} \propto (A_{tot})^{\tau}$$

The role of isospin dependent nuclear charge radii parameterizations on the production of largest fragment increases with increase in A_{tot}.

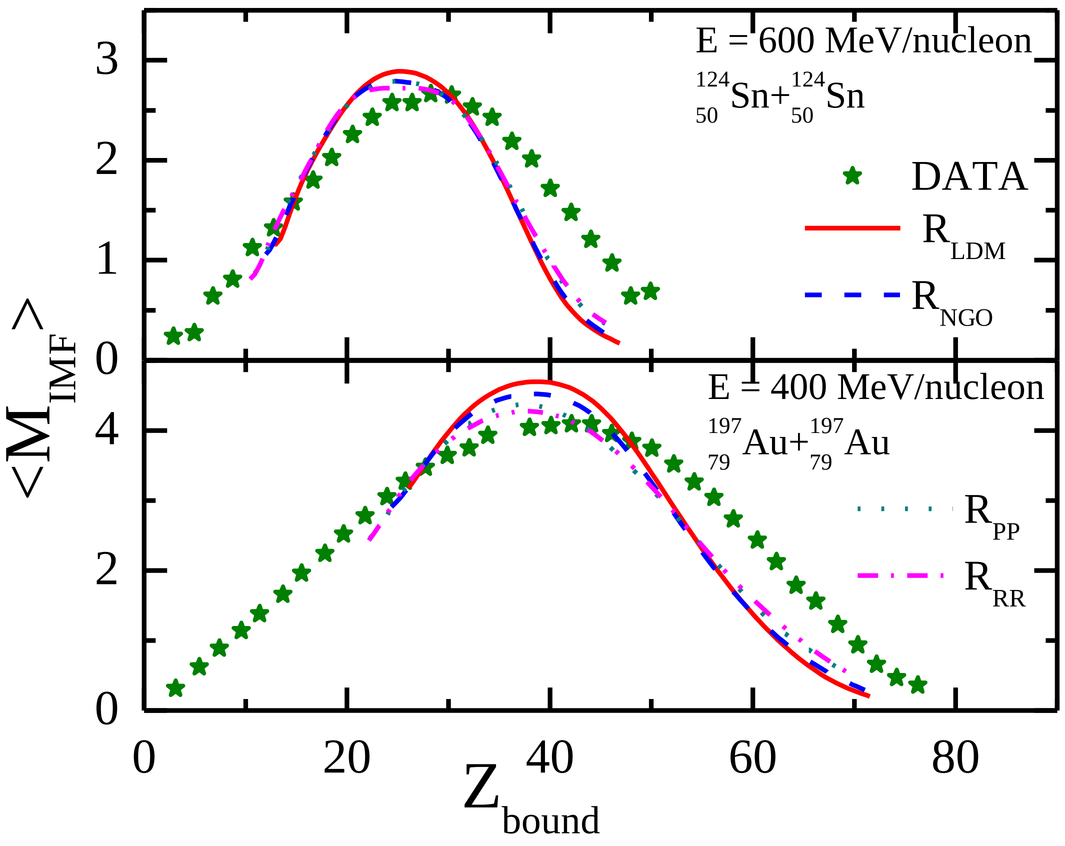
The role of isospin dependent nuclear charge radii parameterizations is dominating at higher colliding geometries.

Sangeeta *et. al.*, Nucl. Phys. A**927**, 220-231 (2014)

Result and Discussion

Structural effects via Nuclear Radius:

Influence of isospin dependent radii on the variation of <M_{IMF} > v/s Z_{bound}



Sangeeta *et. al.*, Nucl. Phys. A**927**, 220-231 (2014)

Result and Discussion

Structural effects via Nuclear Radius:

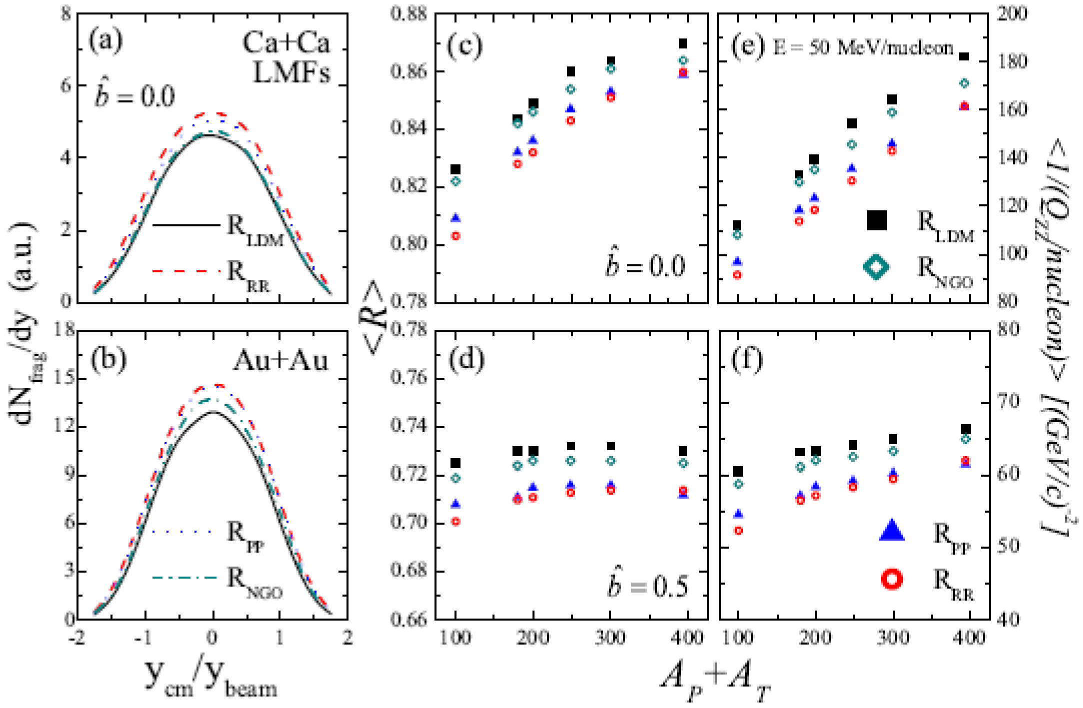


Fig. 1. Panels (a) and (b) display the rapidity distribution of LMFs for central Ca + Ca and Au + Au collisions, respectively, at $E = 50$ MeV/nucleon. Panels (c) and (d) represent $\langle R \rangle$, whereas (e) and (f) represent $\langle 1/(Q_{ZZ}/\text{nucleon}) \rangle$ as a function of system mass in central (upper panels) and semi-central (lower panels) collisions at $E = 50$ MeV/nucleon.

Sangeeta, Acta Phys. Pol. B **47**, 991 (2016)

Result and Discussion

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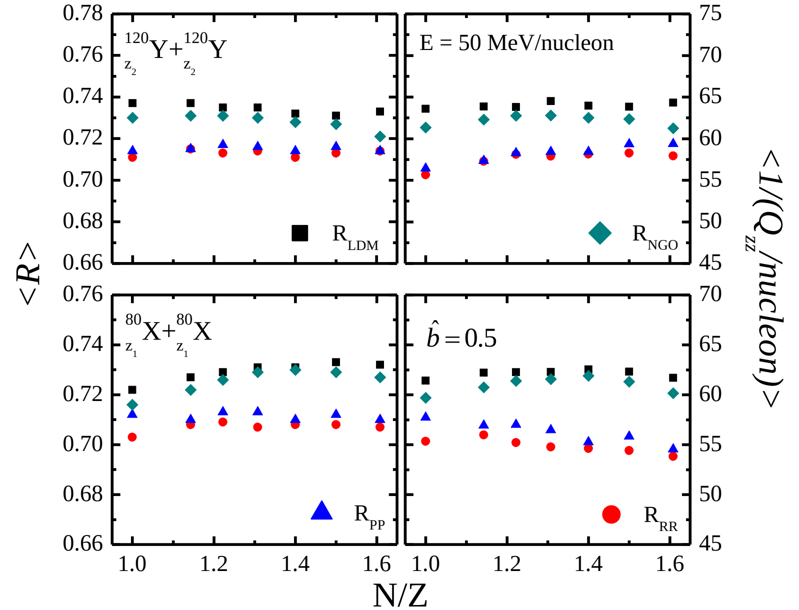
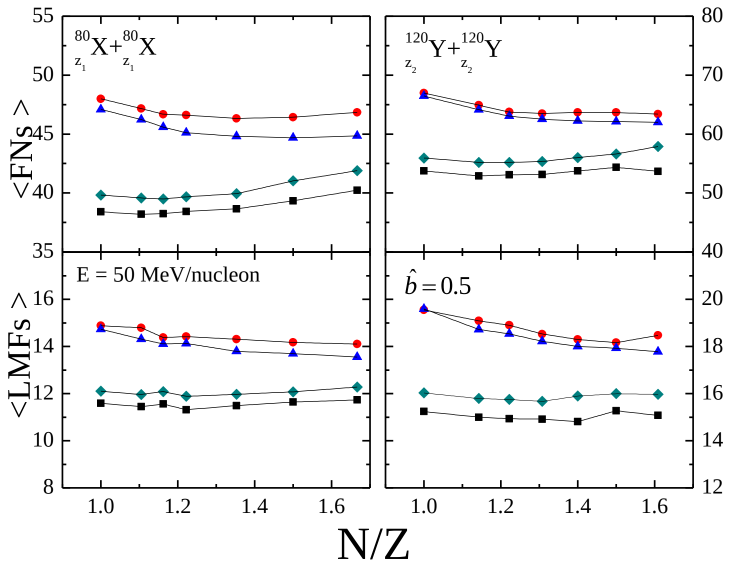
Isospin effects via Nuclear Radius:



Sangeeta, Acta Phys. Pol. B **47**, 991 (2016)



Sangeeta *et. al.*, Nucl. Phys. A**927**, 220-231 (2014)

The behaviour of nuclear stopping as well as fragmentation has been found to be independent towards initial neutron-to-proton ratio of colliding partners with isospin independent nuclear charge radii. Also the effect of isospin content of colliding pair shows no effect on nuclear stopping even if one consider isospin dependent nuclear charge radii parameterization. Whereas, the N/Z dependence of multiplicity of fragments is slightly affected by including isospin dependent nuclear charge radii.

Result and Discussion

- There exist no isospin effects through nulear charge radii on the thermalization.
- I next step, we compared the role of radius in mass symmetric and asymmetric nuclear reactions.

Result and Discussion

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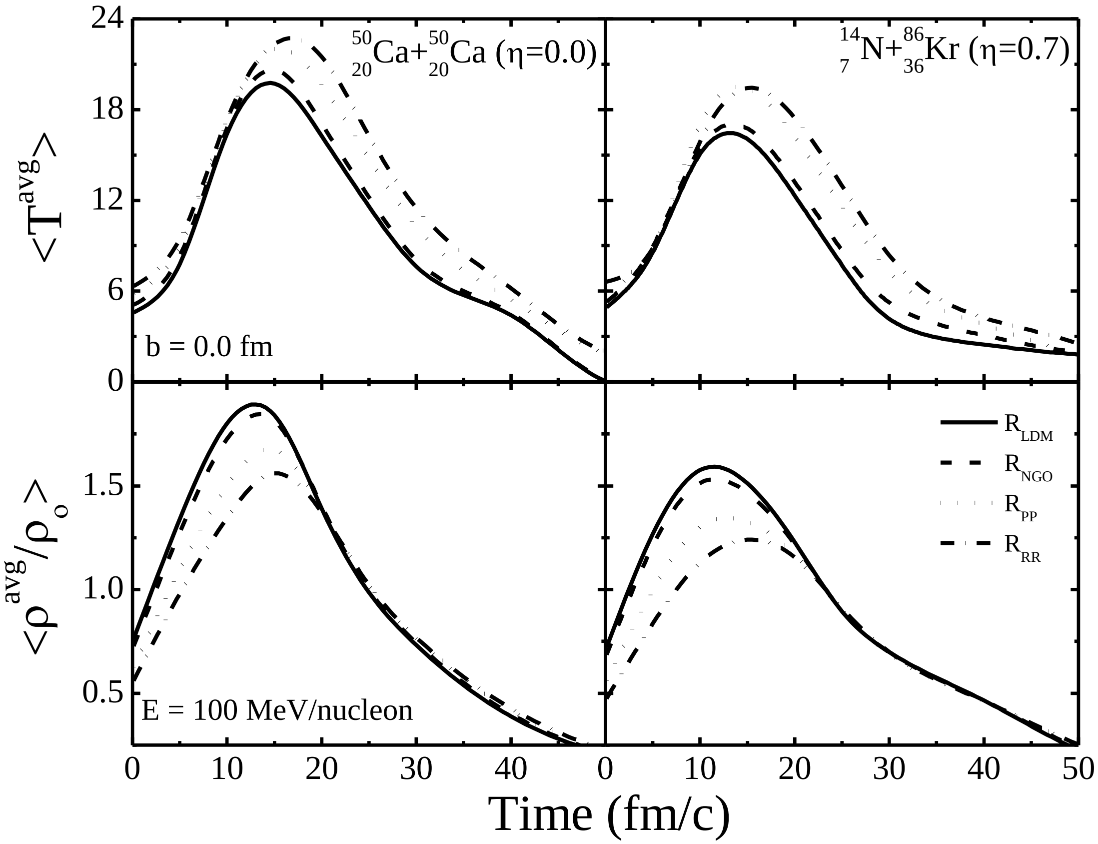
Role of Nuclear Charge Radii Parameterizations on the time evolution of Average Temperature and Average Density:



The mass asymmetry of a reaction can be defined by asymmetry parameter:

$$\eta = \frac{A_T - A_P}{A_T + A_P}$$

A_T : mass of target
A_P :mass of projectile

- η=0; symmetric collisions
- η ≠ 0; asymmetric collisions

Result and Discussion

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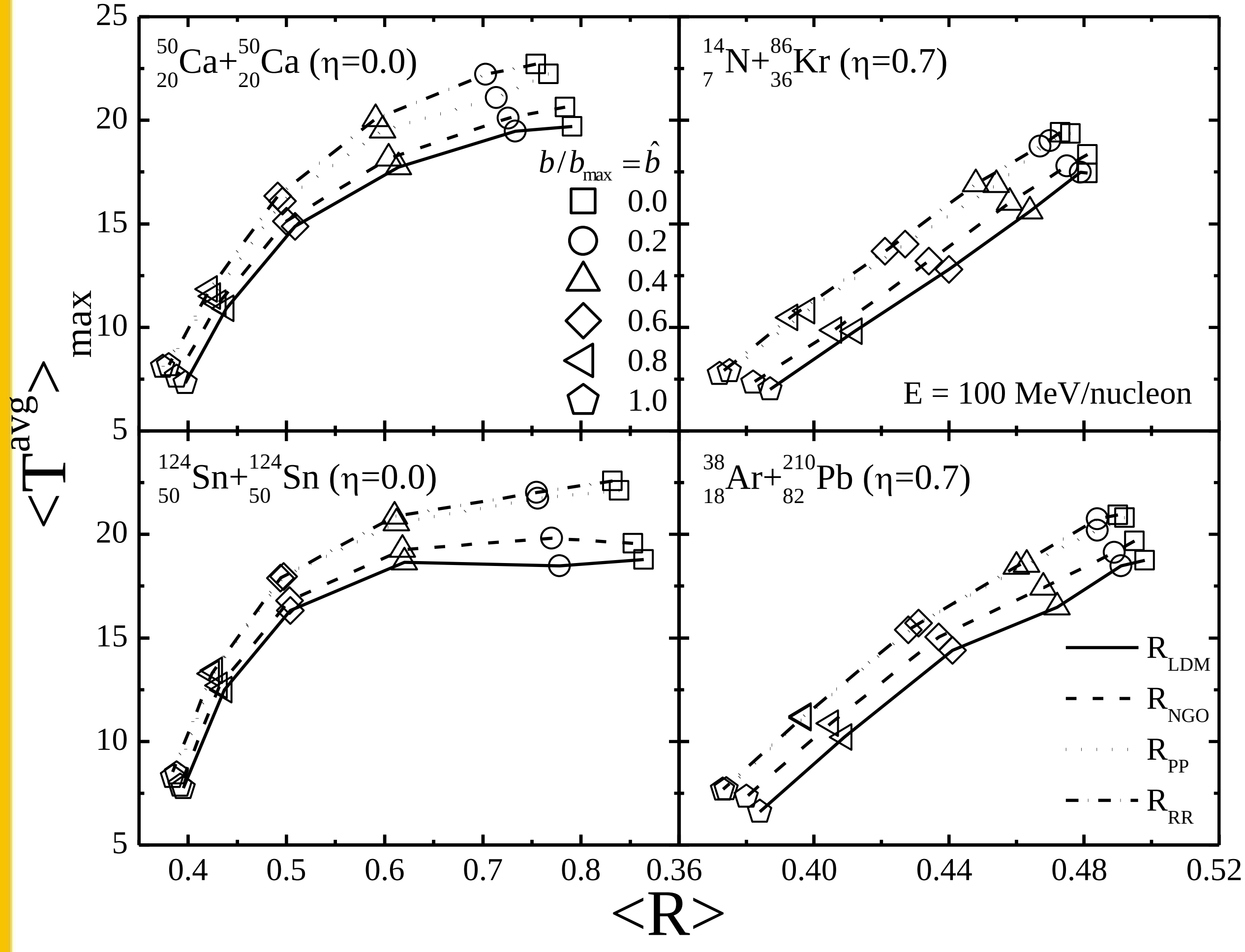
Role of Nuclear Charge Radii Parameterizations on the Correlation between Temperature and Nuclear Stopping:



$$\hat{b} = b / b_{max}$$

$$where\ b_{max} = (R_P + R_T)$$

R_P and R_T are radius of projectile and target respectively.

Conclusion

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For mass symmetric reactions, the curves presenting the correlation, first increases with decrease in ˆ*b and then shows saturation effect.*

Whereas, in mass asymmetric reactions, the curve shows straight line behavior, which means *< Tavg >max and < R > exhibits strong correlation at all* the collision geometries.

The enhanced longitudinal momentum (due to increase in radius) of the particles contributes in raising the temperature of the system.

The ratio of change in radius (while switching from R_{LDM} *to* R_{RR}*) to the* change in *< R > as well as < Tavg >max is more for mass asymmetric reactions (and lighter* reactions) compared to mass symmetric reactions (and heavier reactions). Because of these effects, the curves of correlation are lifted upward as we switch from *RLDM to other three* radii parameterizations.

for symmetric reactions, the role of nuclear charge radii parameterizations on the correlation of temperature and nuclear stopping decreases with decrease in impact parameter; however, it remains almost same for asymmetric reactions.

Acknowledgement

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•Financial grant from Science and Engineering Research Board wide application number **ITS/2349/2016-2017**.

Theoretical Models

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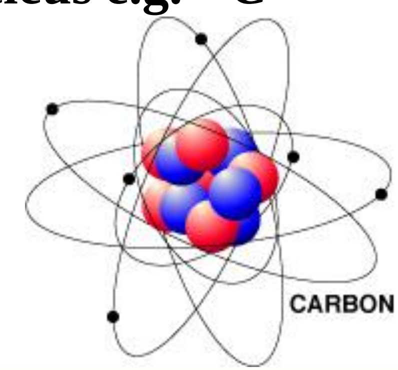
Most of the theoretical approaches use LDM radii parameterization with different values of r_o. $R = r_o A^{1/3}$

Time-Dependent Harteek-Fork (TDHF)	$r_0 = 1.12\ fm$
Inter Nuclear Cascade (INC)	$r_0 = 1.18\ fm$
Quantum Molecular Dynamics (QMD)	$r_0 = 1.142\ fm$
Isospin-dependent Boltzmann-Uehling-Uhlenbeck (IBUU)	$r_0 = 1.12\ fm$
Isospin-dependent Quantum Molecular Dynamics (IQMD)	$r_0 = 1.12\ fm$

Some models preferred to use rms radii of particular nucleus e.g. ^{12}C

Fermionic Molecular Dynamics (FMD)	**2.79 fm**
Antisymmetric Molecular Dynamics (AMD)	**2.49 fm**
nucleus-nucleus optical potential models	**2.35 fm**



Methodology

Potentials in IQMD model:

The baryon-baryon potential is given by: $V^{ij} = V^{ij}_{Skyrme} + V^{ij}_{Yukawa} + V^{ij}_{Coul} + V^{ij}_{mdi} + V^{ij}_{sym}$

Skyrme potential

$$V^{ij}_{Skyrme} = \frac{\alpha}{2}\left(\frac{\rho}{\rho_o}\right) + \frac{\beta}{\gamma+1}\left(\frac{\rho}{\rho_o}\right)^{\gamma}$$

Yukawa potential

$$V^{ij}_{Yukawa} = \sum_{j,i\neq j} t_3 \frac{\exp\left\{-\left|r_i - r_j\right|\right\}/\mu}{-\left|r_i - r_j\right|/\mu}$$

Coulomb potential

$$V^{ij}_{Coul} = \sum_{j,i\neq j} \frac{Z_i Z_j e^2}{\left|r_i - r_j\right|}$$

Symmetry potential

$$V^{ij}_{Sym} = t_6 \frac{1}{\rho_0} T_{3i} T_{3j} \delta(r_i - r_j)$$

Skyrme, Yukawa → **Isospin independent**

Coulomb, Symmetry → **Isospin dependent**

$$V^{ij}_{mdi} = t_4 \ln^2[t_5(\overrightarrow{p'_i} - \overrightarrow{p})^2 + 1](\overrightarrow{r'} - \overrightarrow{r})$$

Momentum –dependent interactions → Isospin independent

$$\sigma_{np} = 3\,(\sigma_{nn} \text{ or } \sigma_{pp})$$

nn cross section → Isospin dependent