Global phenomenological and microscopic optical potentials for alpha

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I. Introduction

- **The optical model is one of the most basic theoretical models in nuclear reaction theory. The key of optical model is the optical model potential, which includes the phenomenological and microscopic optical potential.**

- **The microscopic optical potential is achieved on the basis of many-body theory and nucleon–nucleon interaction. The phenomenological optical potential is obtained on the basis of the available nuclear reaction experimental data.**

- **The microscopic and global phenomenological optical potentials for alpha incident are presented in this work.**

Ⅱ. Theory and formulation

1. Microscopic optical potential (MOP)

The Hamiltonian composed of the two-body interaction can be written as:

$$H = H_0 + H_1 \qquad (2.1)$$

where,

$$H_0 = \sum_i (t_i + U_i),$$

$$H_1 = \frac{1}{2}\sum_{i\neq j} V_{ij} - \sum_i U_i$$

H_0 is the single-particle Hamiltonian, H_1 is the residual interaction, and U_i is the mean field of the single-particle.

The expression of four-particle Green function:

$$
\begin{aligned}
& iG(\alpha_1\alpha_2\alpha_3\alpha_4, \beta_1\beta_2\beta_3\beta_4; t_1 - t_2) \\
& = \frac{\left\langle \phi_0 \middle| T[U_\eta(\infty, -\infty)\xi_{\alpha_4}(t_1)\xi_{\alpha_3}(t_1)\xi_{\alpha_2}(t_1)\xi_{\alpha_1}(t_1)\xi^+_{\beta_1}(t_2)\xi^+_{\beta_2}(t_2)\xi^+_{\beta_3}(t_2)\xi^+_{\beta_4}(t_2)] \middle| \phi_0 \right\rangle}{\left\langle \phi_0 \middle| U_\eta(\infty, -\infty) \middle| \phi_0 \right\rangle} \\
& = \left\langle \phi_0 \middle| T[U_\eta(\infty, -\infty)\xi_{\alpha_4}(t_1)\xi_{\alpha_3}(t_1)\xi_{\alpha_2}(t_1)\xi_{\alpha_1}(t_1)\xi^+_{\beta_1}(t_2)\xi^+_{\beta_2}(t_2)\xi^+_{\beta_3}(t_2)\xi^+_{\beta_4}(t_2)] \middle| \phi_0 \right\rangle_L
\end{aligned}
\tag{2.2}
$$

$$
\begin{aligned}
iG(\alpha_1\alpha_2\alpha_3\alpha_4, \beta_1\beta_2\beta_3\beta_4; t_1 - t_2) = {} & iG^{(0)}(\alpha_1\alpha_2\alpha_3\alpha_4, \beta_1\beta_2\beta_3\beta_4; t_1 - t_2) \\
& + iG^{(1)}(\alpha_1\alpha_2\alpha_3\alpha_4, \beta_1\beta_2\beta_3\beta_4; t_1 - t_2) \\
& + iG^{(2)}(\alpha_1\alpha_2\alpha_3\alpha_4, \beta_1\beta_2\beta_3\beta_4; t_1 - t_2) + \cdots\cdots
\end{aligned}
\tag{2.3}
$$

It satisfies the Dyson equation:

$$
\begin{aligned}
iG(\alpha_1\alpha_2\alpha_3\alpha_4, \beta_1\beta_2\beta_3\beta_4; \omega) = {} & iG^{(0)}(\alpha_1\alpha_2\alpha_3\alpha_4, \beta_1\beta_2\beta_3\beta_4; \omega) + \frac{i}{\hbar} \sum_{\rho\lambda\theta\varepsilon\mu\nu\delta\gamma} iG^{(0)}(\alpha_1\alpha_2\alpha_3\alpha_4, \rho\lambda\theta\varepsilon; \omega) \\
& \bullet [U_{\rho\lambda\theta\varepsilon, \mu\nu\delta\gamma} - M(\rho\lambda\theta\varepsilon, \mu\nu\delta\gamma; \omega)] iG(\mu\nu\delta\gamma, \beta_1\beta_2\beta_3\beta_4; \omega)
\end{aligned}
\tag{2.4}
$$

The first-order mass operator which is used to represent the real part of the MOP is expressed as

$$M^{(1)}_{\alpha_1\alpha_2\alpha_3\alpha_4} = \sum_{\rho} V_{\alpha_1\rho,\alpha_1\rho} n_\rho + \sum_{\rho} V_{\alpha_2\rho,\alpha_2\rho} n_\rho + \sum_{\rho} V_{\alpha_3\rho,\alpha_3\rho} n_\rho + \sum_{\rho} V_{\alpha_4\rho,\alpha_4\rho} n_\rho \qquad (2.7)$$

Real part of nucleon MOP

The imaginary part of the second-order mass operator which represent the imaginary part of MOP is expressed as

$$\begin{aligned}
\mathrm{Im}\, M^{(2)}_{\alpha_1\alpha_2\alpha_3\alpha_4}(E) = & -\frac{\pi}{2} \sum_{\rho\delta\lambda} V_{\alpha_1\lambda,\delta\rho} V_{\delta\rho,\alpha_1\lambda} n_\lambda (1-n_\delta)(1-n_\rho)\delta(\varepsilon_{\alpha_1} - \varepsilon_\rho - \varepsilon_\delta + \varepsilon_\lambda) \\
& -\frac{\pi}{2} \sum_{\rho\delta\lambda} V_{\alpha_2\lambda,\delta\rho} V_{\delta\rho,\alpha_2\lambda} n_\lambda (1-n_\delta)(1-n_\rho)\delta(\varepsilon_{\alpha_2} - \varepsilon_\rho - \varepsilon_\delta + \varepsilon_\lambda) \\
& -\frac{\pi}{2} \sum_{\rho\delta\lambda} V_{\alpha_3\lambda,\delta\rho} V_{\delta\rho,\alpha_3\lambda} n_\lambda (1-n_\delta)(1-n_\rho)\delta(\varepsilon_{\alpha_3} - \varepsilon_\rho - \varepsilon_\delta + \varepsilon_\lambda) \qquad (2.8) \\
& -\frac{\pi}{2} \sum_{\rho\delta\lambda} V_{\alpha_4\lambda,\delta\rho} V_{\delta\rho,\alpha_4\lambda} n_\lambda (1-n_\delta)(1-n_\rho)\delta(\varepsilon_{\alpha_4} - \varepsilon_\rho - \varepsilon_\delta + \varepsilon_\lambda) \\
& = \mathrm{Im}\, M^{(2)}_{\alpha_1}\left(\frac{E}{4}\right) + \mathrm{Im}\, M^{(2)}_{\alpha_2}\left(\frac{E}{4}\right) + \mathrm{Im}\, M^{(2)}_{\alpha_3}\left(\frac{E}{4}\right) + \mathrm{Im}\, M^{(2)}_{\alpha_4}\left(\frac{E}{4}\right)
\end{aligned}$$

Imaginary part of nucleon MOP

When the distribution of the proton and neutron in alpha is considered, the MOP for alpha can be obtained by folding the microscopic optical potentials of its constituent nucleons in the ground state of alpha

$$V_\alpha(R) = \iiint \left|\psi(\vec{\xi}_1, \vec{\xi}_2, \vec{\xi}_3)\right|^2 \Big[V_n(\vec{R} + \frac{1}{2}\vec{\xi}_1 + \frac{1}{2}\vec{\xi}_3) + V_n(\vec{R} - \frac{1}{2}\vec{\xi}_1 + \frac{1}{2}\vec{\xi}_3) \qquad (2.9)$$

$$+ V_p(\vec{R} + \frac{1}{2}\vec{\xi}_2 - \frac{1}{2}\vec{\xi}_3) + V_p(\vec{R} - \frac{1}{2}\vec{\xi}_2 - \frac{1}{2}\vec{\xi}_3) \Big] d\vec{\xi}_1 d\vec{\xi}_2 d\vec{\xi}_3$$

where,

$$\psi(\vec{\xi}_1, \vec{\xi}_2, \vec{\xi}_3) = \left(\frac{\beta^3}{4\pi^3}\right)^{\frac{3}{4}} e^{-\frac{1}{4}\beta(\xi_1{}^2 + \xi_2{}^2) - \frac{1}{2}\beta\xi_3{}^2}, \quad \beta = 0.4395. \qquad (2.10)$$

V_n and V_p are the MOPs for neutron and proton, which are obtained [1] by calculating the mass operator of one-particle Green function based on the Skyrme interaction (GS2) using nuclear matter approximation and local density approximation.

[1] Q. Shen, Y. Han, and H. Guo, Phys. Rev. C 80, 024604 (2009).

2. Global phenomenological optical potential (GOP)

- **The GOP is obtained by adjusting its parameters to fit the existing experimental data of reaction cross sections and elastic scattering angular distributions.**

The GOP is expressed as

$$U(r,E) = -V_r(E) f_r(r) - i[4a_s W_s(E)\frac{df_s(r)}{dr} + W_v(E_L) f_v(r)] + V_C(r),$$

with Woods-Saxon factor as

$$f_i(r) = \left[1 + \exp((r - r_i A^{\frac{1}{3}}) \big/ a_i)\right]^{-1} \qquad i = r,\ v,\ s$$

The potential depths are expressed as

$$V_r(E) = V_0 + V_1 E + V_2 E^2 + V_3 (N-Z)/A + V_4 Z/A^{1/3}$$

$$W_s(E) = Max\{0, W_0 + W_1 E + W_2 (N-Z)/A\}$$

$$W_v(E) = Max\{0, U_0 + U_1 E\}$$

Parameters of the global phenomenological optical potential

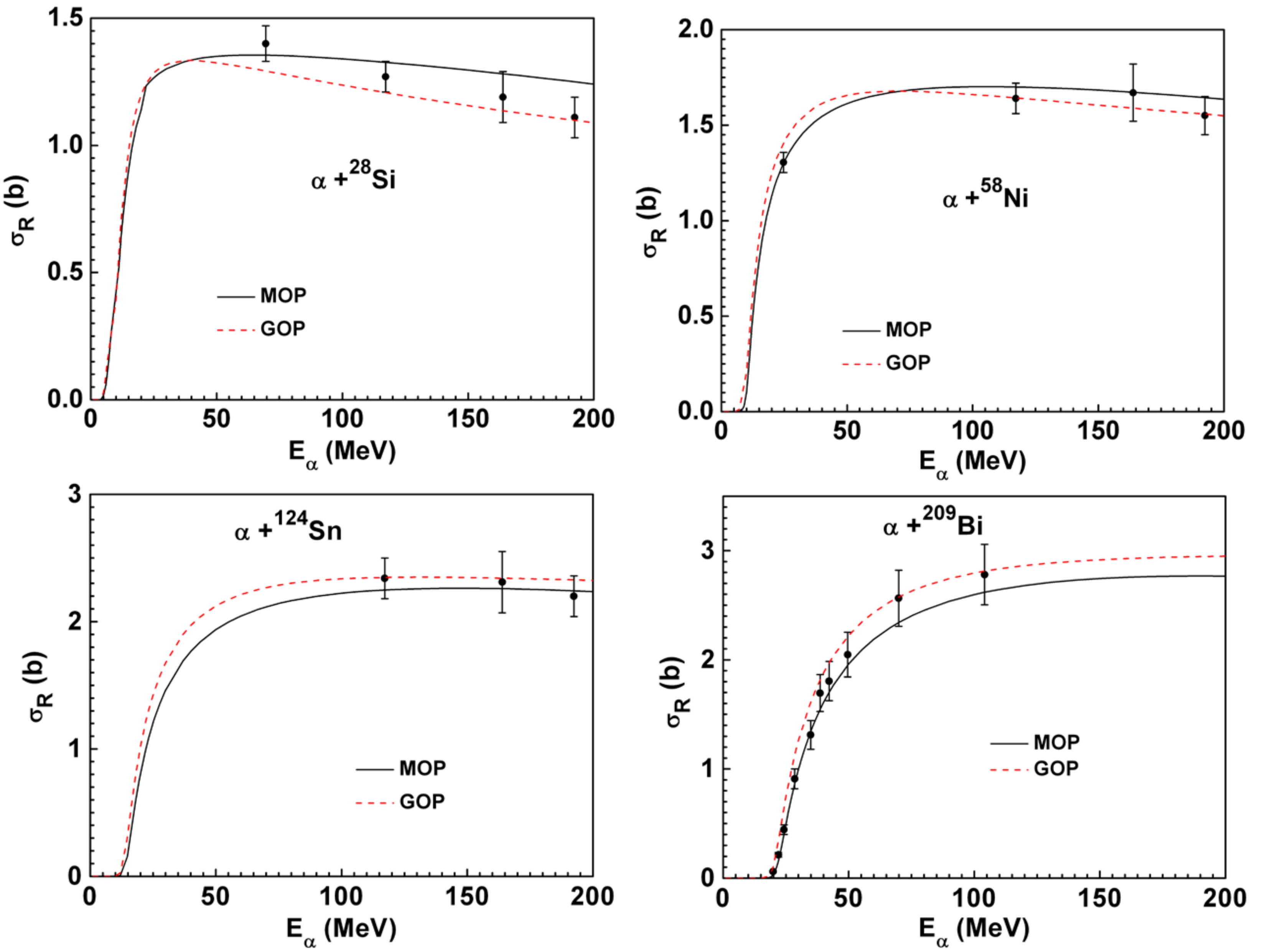
The experimental data of elastic scattering angular distributions from different experiment have some divergence for the same target. We analyze these results using the obtained alpha MOP. The experimental data are chosen and applied to achieve the GOP.

V_0 (MeV)	175.0881	U_1 (MeV)	0.1409
V_1 (MeV)	-0.6236	r_C (fm)	1.35
V_2 (MeV)	-0.0006	r_r (fm)	1.3421
V_3 (MeV)	30.0	r_s (fm)	1.2928
V_4 (MeV)	-0.236	r_v (fm)	1.4259
W_0 (MeV)	27.5816	a_r (fm)	0.6578
W_1 (MeV)	-0.0797	a_s (fm)	0.6359
W_2 (MeV)	48.0	a_v (fm)	0.5587
U_0 (MeV)	-4.0174		

III. Results and analyses

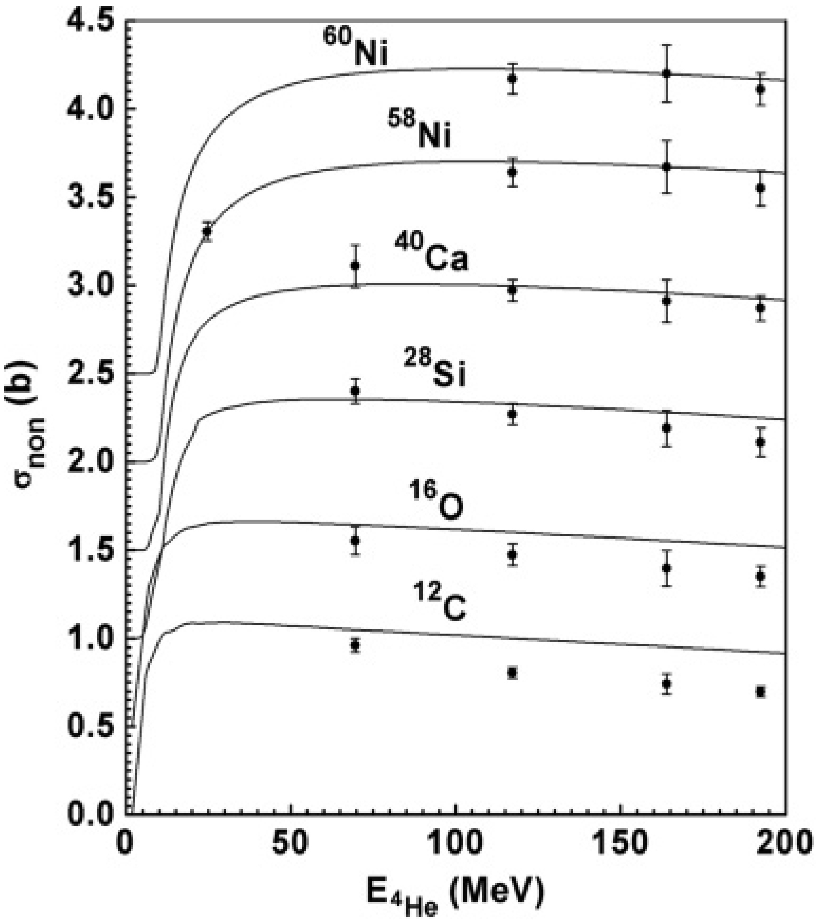
The reaction cross sections and elastic scattering angular distributions for alpha-induced reactions are calculated by the MOP and the GOP in the target nucleus mass range $12 \leq A \leq 209$ at incident energies below 400 MeV, the theoretical results are compared with the experimental data.

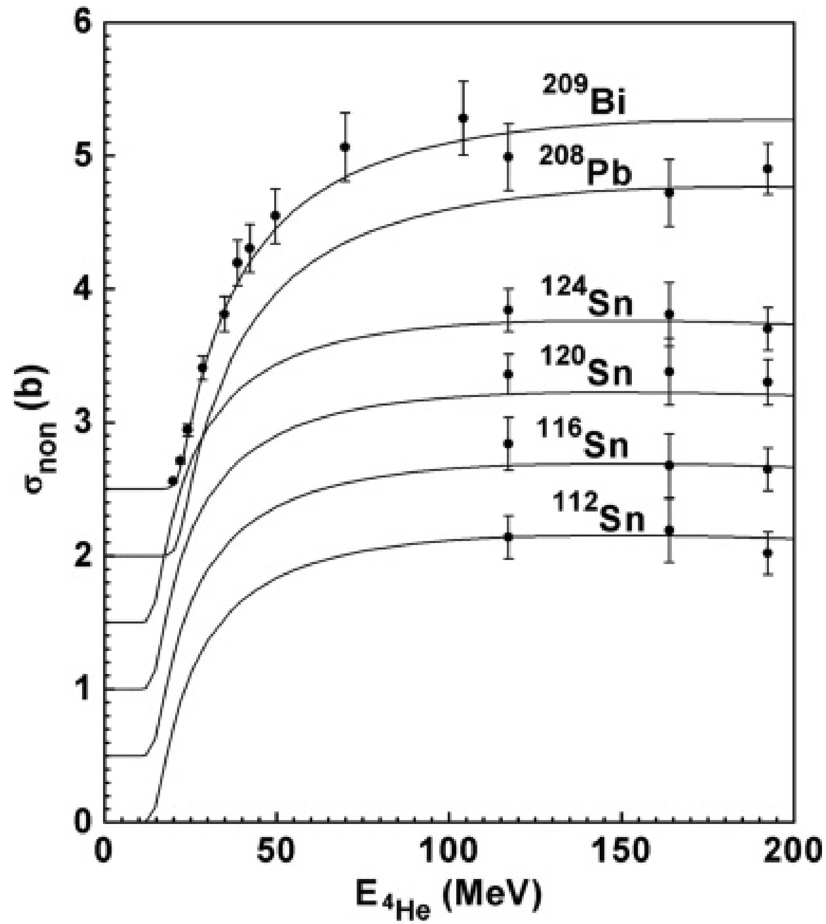
✓ Reaction cross sections



✓ Reaction cross sections

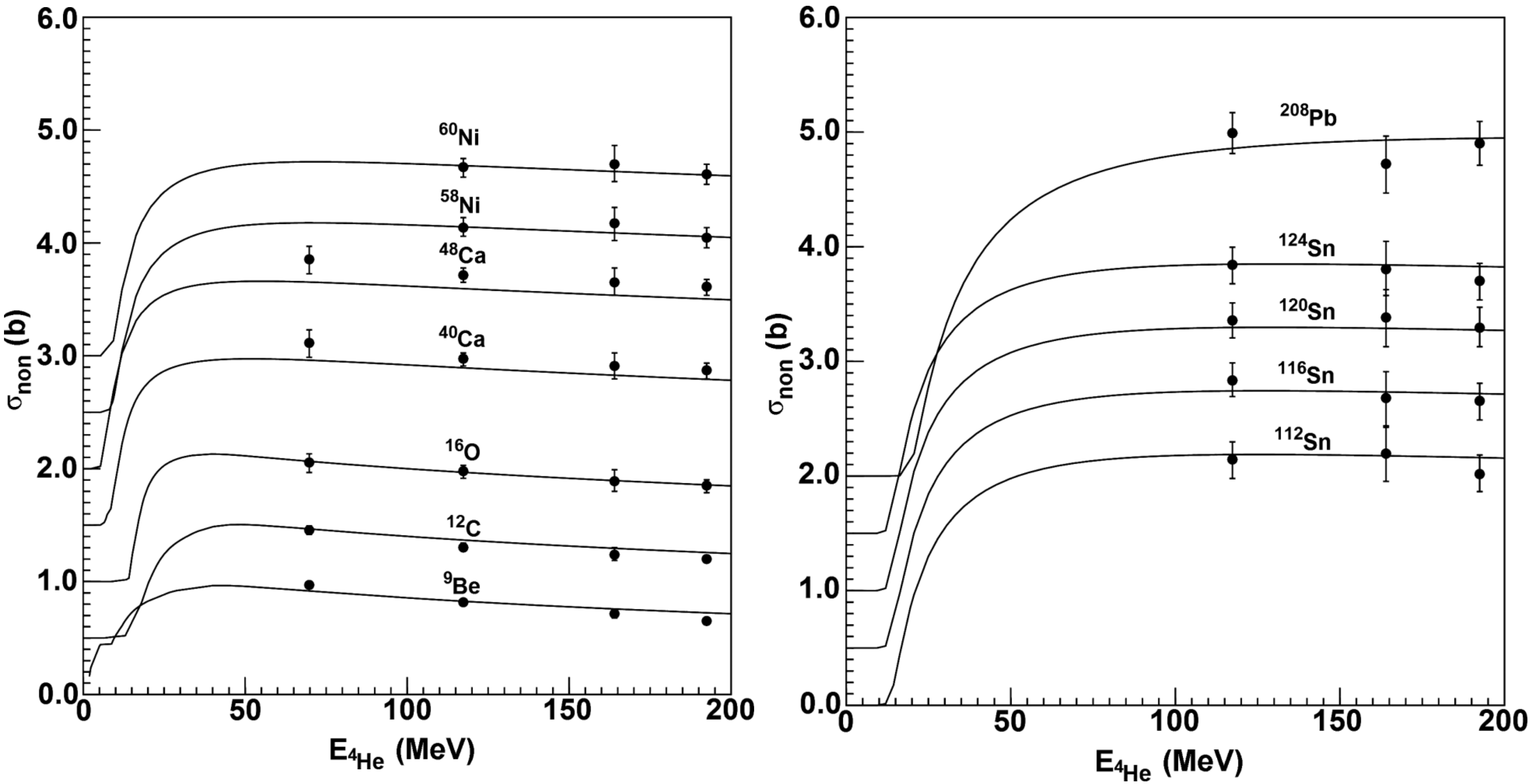
The results of MOP



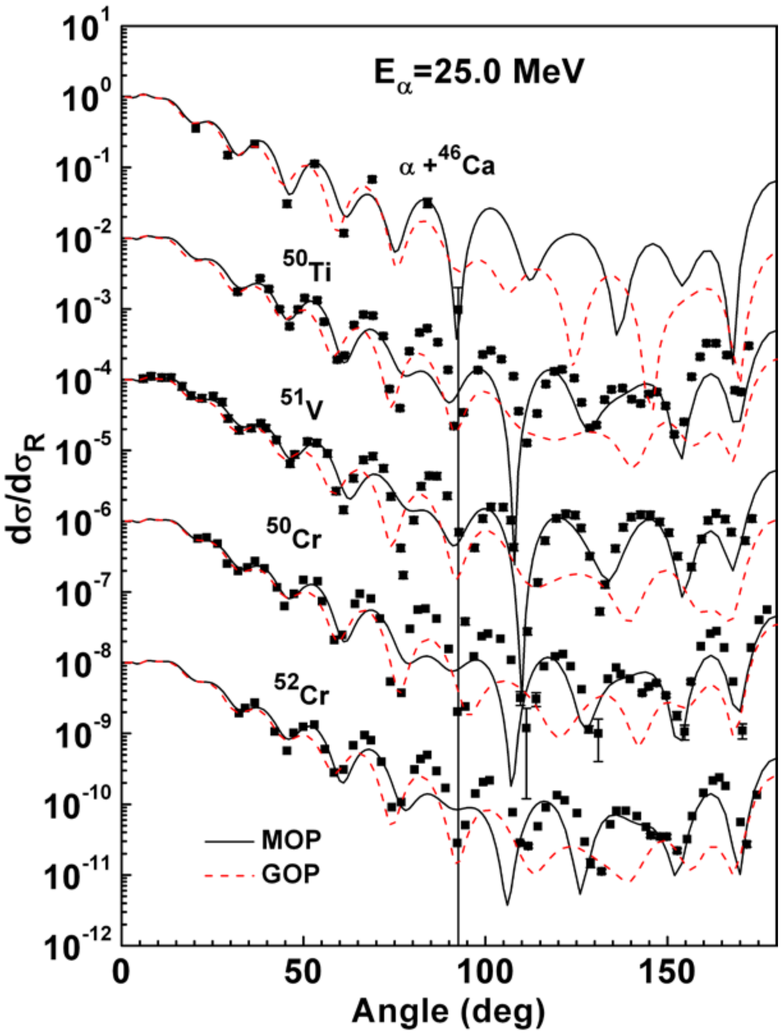


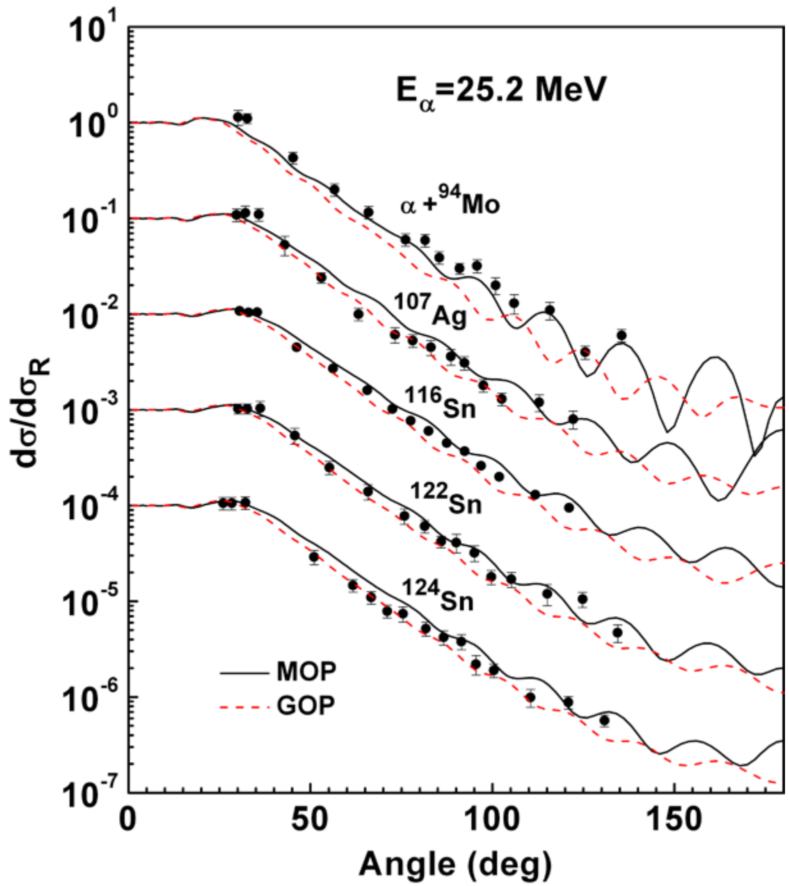
✓ Reaction cross sections

The results of GOP

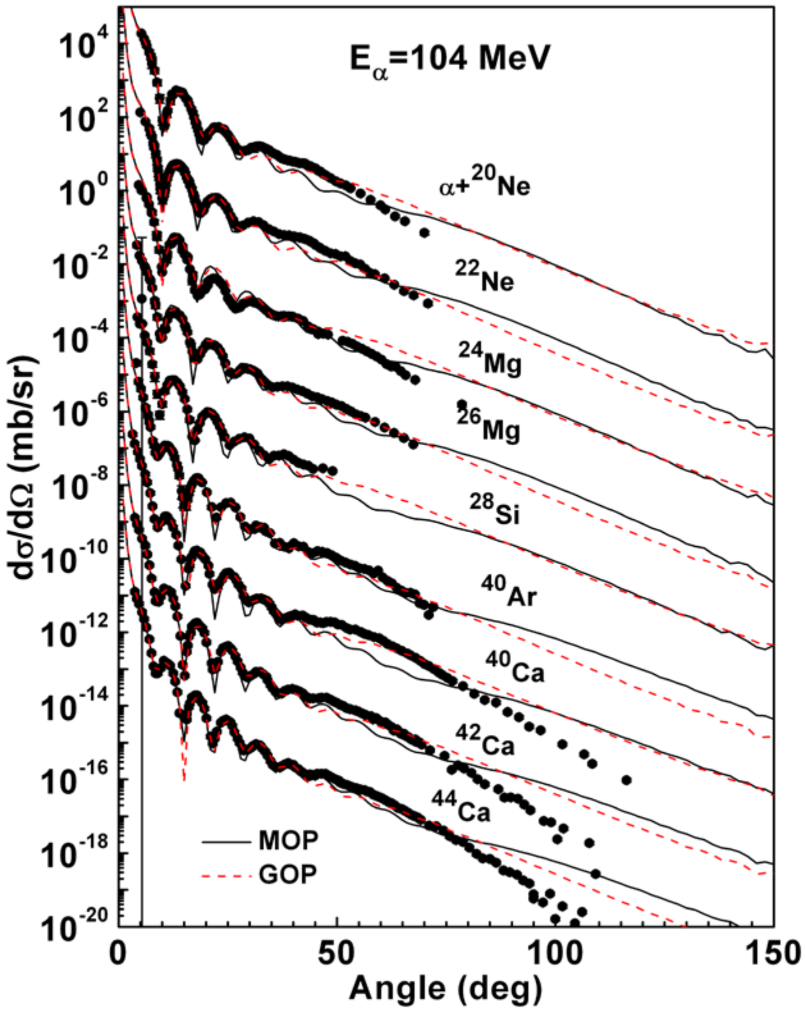


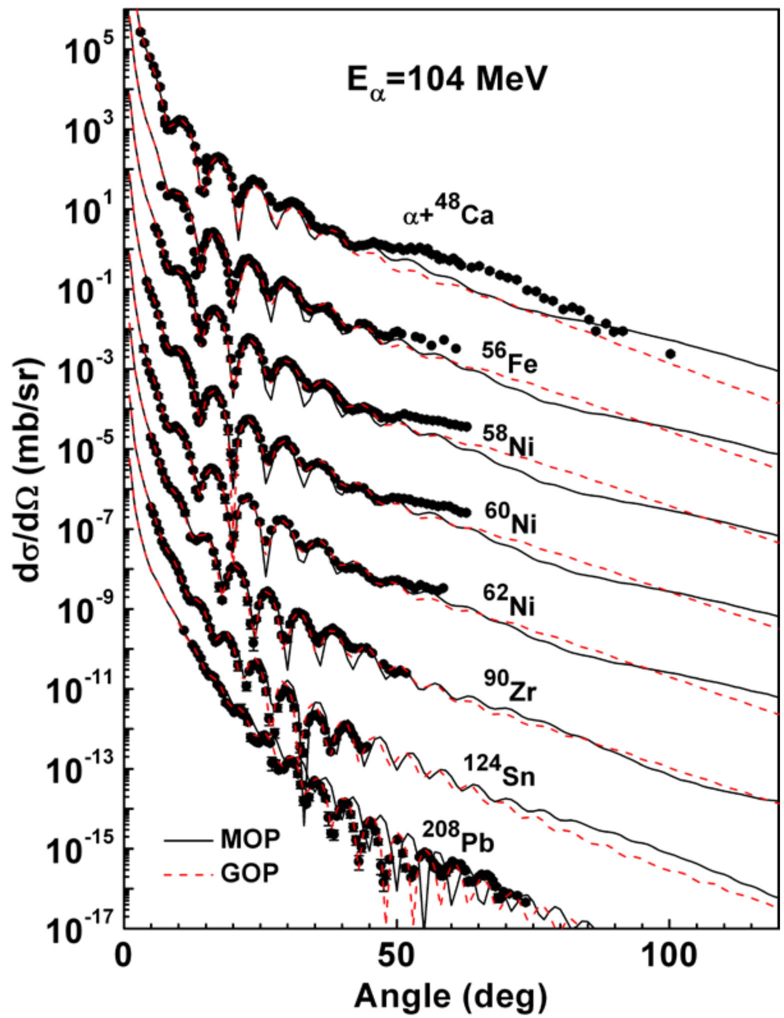
✓ Alpha elastic scattering angular distribution



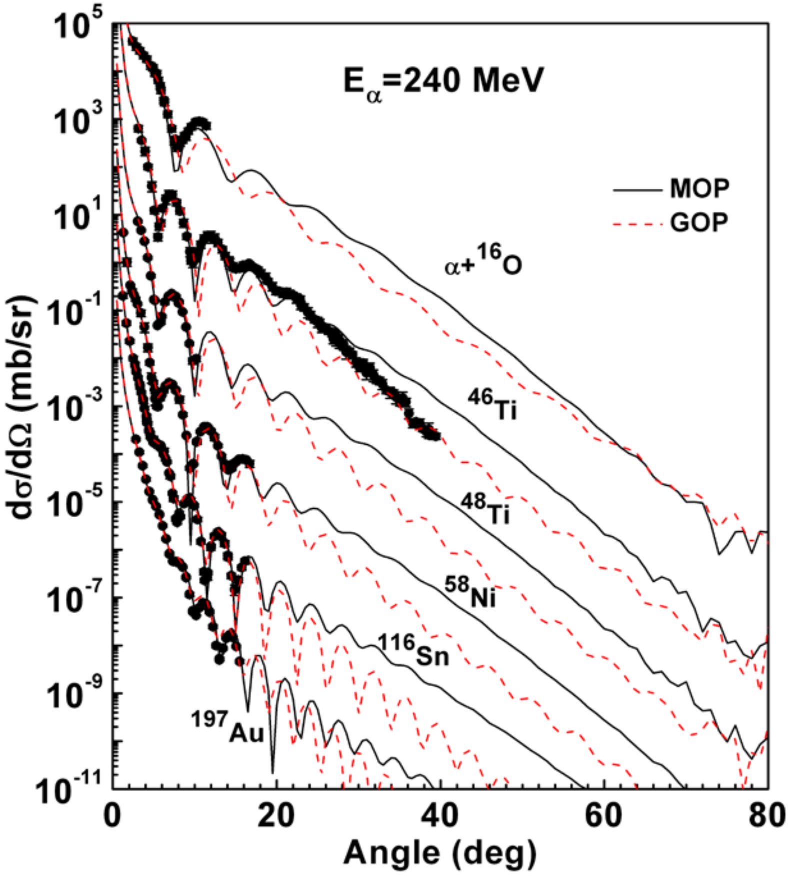


✓ Elastic scattering angular distribution

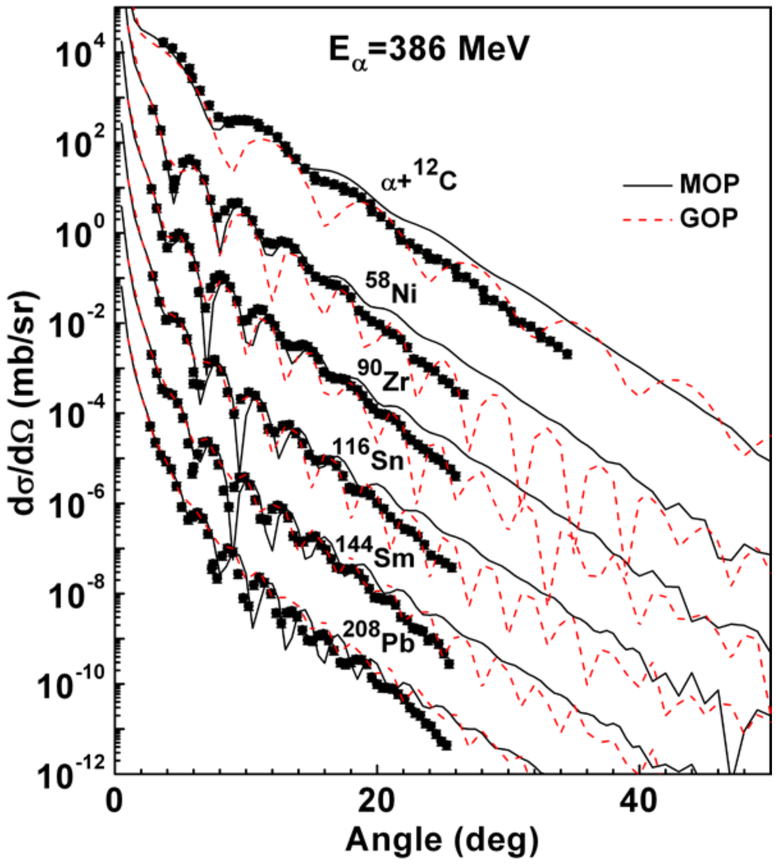




✓ Elastic scattering angular distribution

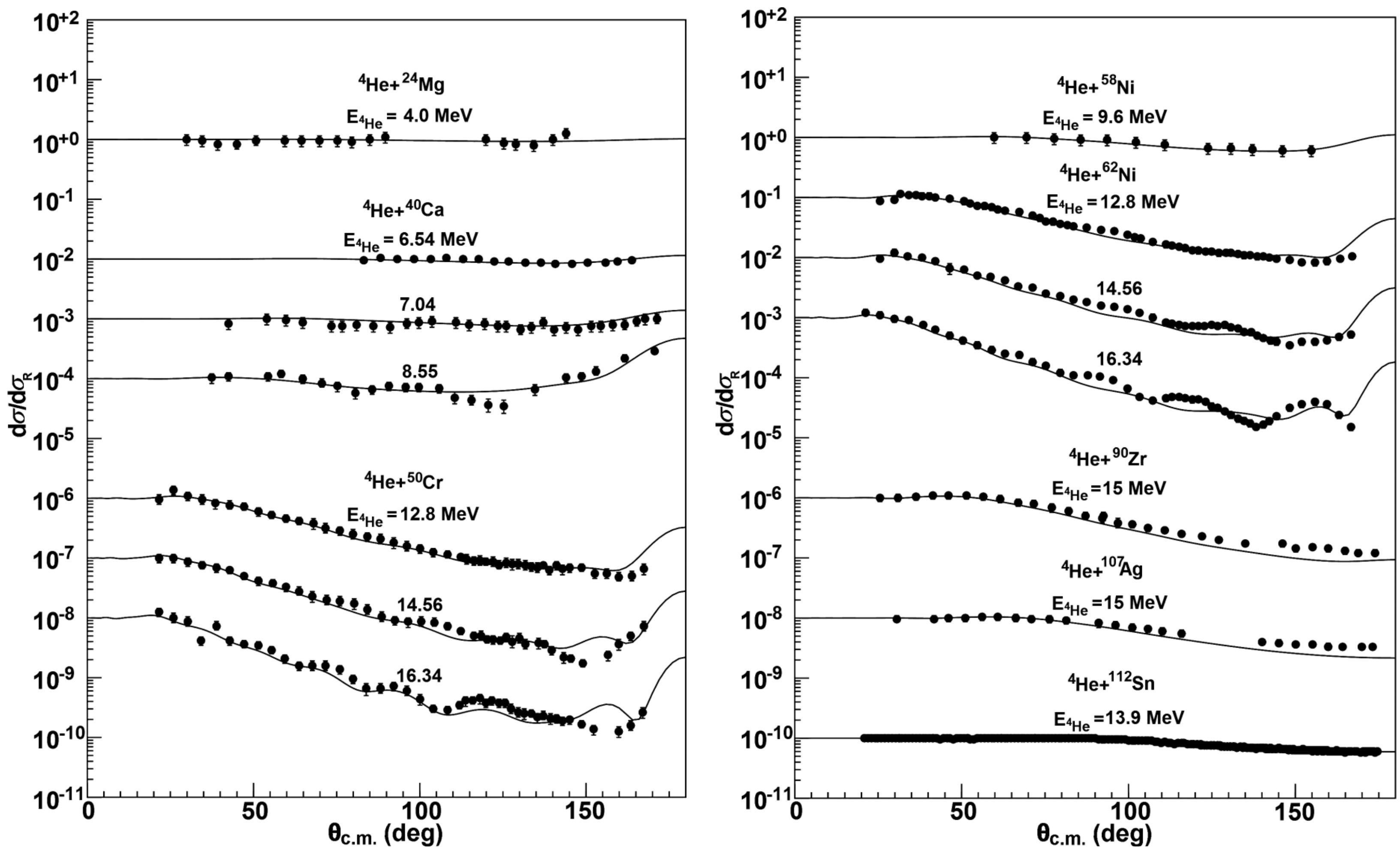


✓ **Elastic scattering angular distribution**

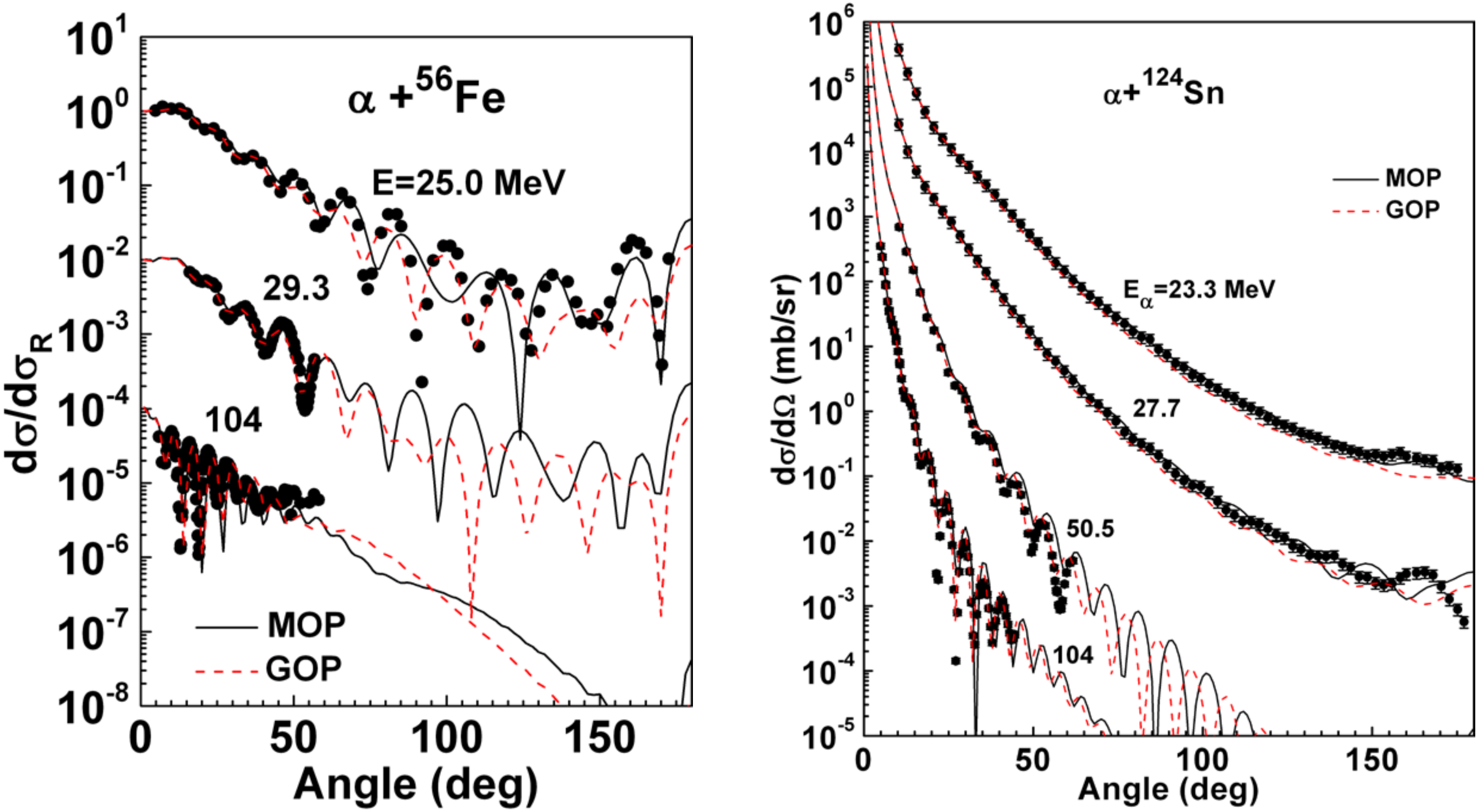


✓ Alpha elastic scattering angular distribution

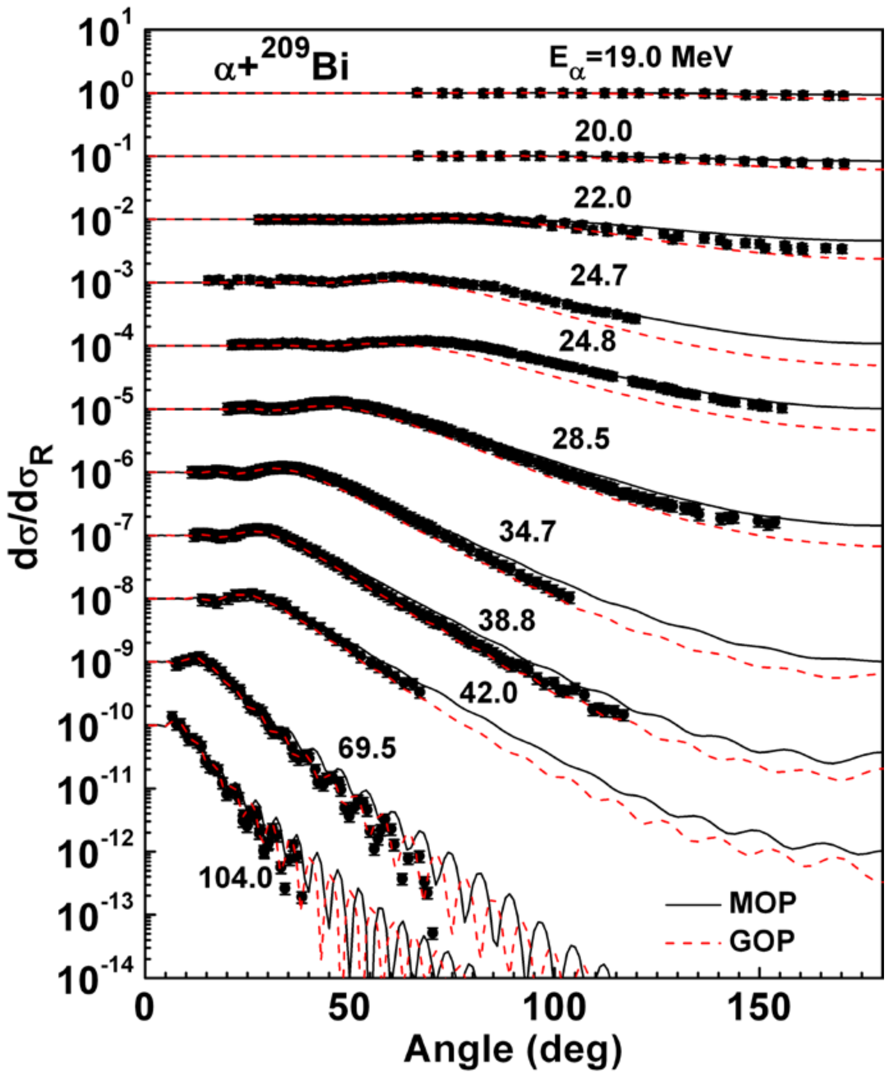
The results of GOP at low incident energies



✓ **Elastic scattering angular distribution**



✓ Elastic scattering angular distribution



IV. Summary

1. An alpha microscopic optical potential is obtained by the Green function method through nuclear matter approximation and local density approximation based on the Skyrme interactions.

2. An alpha Global phenomenological optical potential is obtained by adjusting its parameters to fit the existing experimental data.

3. The calculated results of reaction cross section and elastic scattering angular distribution with MOP and GOP are generally in reasonable agreement with the experimental data. In most cases, the GOP is better than the MOP in reproducing the experimental data.

Thank you!

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